Use of entangled coherent states in quantum teleportation and entanglement diversion

Ranjana Prakash*
Physics Department, University of Allahabad, Allahabad-211002, India

Van Enk and Hirota [S J Van Enk, and O Hirota, Phys. Rev. A **64**, 022313 (2001)] showed that standard quantum teleportation of one qubit of information encoded in a single mode superposition of coherent states $|\pm\alpha\rangle$ is possible using resource of a two mode entangled coherent state (ECS) with success equal to 1/2. Also, Wang considered [X Wang, Phys. Rev. A **64**, 022302 (2001)] teleportation of a single qubit encoded on bipartite SCS using three-partite ECS and reported success of 1/2. I present here the work done in collaboration with Hari Prakash, Naresh Chandra and Shivani, modifying the van Enk Hirota scheme so as to obtain teleportation with near perfect success and fidelity. I also present our work on controlled quantum teleportation using four-partite entangled state.

If Alice is connected to Bob and also to Charlie with shared bipartite ECS's, she can communicate with Bob and Charlie but Bob and Charlie cannot communicate with each other. I discuss our work on a near perfect scheme generating entanglement of the qubits with Bob and Charlie, called entanglement diversion, and show how it can be achieved with almost perfect fidelity and success.

In this talk I propose to share some of the work done in collaboration with Prof. Hari Prakash, Prof. Naresh Chandra and Dr. Shivani A. Kumar on use of entangled coherent states in quantum teleportation and entanglement diversion.

In classical information theory, unit of classical information is a bit (values 0 and 1) which describe state of a classical two level system and classical information consists of a string of bits (0's and 1's). In quantum mechanics, however, a two-level system can exist not only in states $|0\rangle$ and $|1\rangle$ but also in an infinity of superposition states $|\psi\rangle = a|0\rangle + b|1\rangle$, where a and b are arbitrary complex numbers which satisfy the normalization condition, $\langle\psi|\psi\rangle = |a|^2 + |b|^2 = 1$. Without reducing generality we can take a real and write $a = \cos\frac{\theta}{2}, b = \sin\frac{\theta}{2}\, e^{\phi}$, where the angles θ, ϕ are in the ranges $0 \le \theta \le \pi, 0 \le \phi < 2\pi$. Obviously these ranges associate a direction with each value of (θ, ϕ). Bloch sphere is the name given to a unit sphere, the point $(1, \theta, \phi)$ on which represents the state $|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}\, e^{\phi}|1\rangle$. Unit of information in quantum information theory is a qubit represented by the above-given general state $|\psi\rangle$. For 3, 4 and in general d dimensional states, we define the units of quantum information Qutrit, Ququat and Qudit.

A two qubit pure state may, in special cases, be factorizable, e.g., $|\psi\rangle = |\psi_1\rangle_1 \otimes |\psi_2\rangle_2$. If it is not so it is called entangled. A mixed two qubit state can always be expressed by a density operator of the form, $\rho = \sum_i \rho_i |\psi_i\rangle\langle\psi_i|$, where $\rho_i \ge 0$. If all states $|\psi_i\rangle$ are factorizable, the state is called separable. If the state is not separable it is called entangled.

Entanglement is quantified and is described by parameters, entanglement of formation, concurrence, negativity, quantum discord, three tangle and partial tangle. We shall however use only concurrence. For a pure state $|\psi\rangle$, entanglement is defined by $C(|\psi\rangle) = |\langle\psi|\tilde{\psi}\rangle|, |\tilde{\psi}\rangle = \sigma_y \otimes \sigma_y |\psi^*\rangle$. For a mixed state described by density operator ρ, one defines $\tilde{\rho} = \sigma_y \otimes \sigma_y \rho^* \sigma_y \otimes \sigma_y$. If $\lambda_{1,2,3,4}$ are positive square roots of eigenvalues of $\rho\tilde{\rho}$, concurrence is given by $C(\rho) = max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$.

Quantum Teleportation (QT) is an important quantum information process where a quantum state of a system is transferred on to another similar system at a distance without sending directly any part of the system or of the information about the system. The first protocol for QT was given by Bennett et al [1], for sending one qubit information which is quantum state $|I\rangle = a|0\rangle + b|1\rangle$ of some particle from a sender, say, Alice, to a receiver, say, Bob. Two entangled qubits are shared between Alice and Bob, Alice performs Bell State Measurement (BSM) on the two qubits with her and conveys her result to Bob through a classical channel. Bob then performs a unitary transformation on his qubit dependent on the information he receives from Alice and Bob's qubit assumes the original quantum state of the Alice's information qubit. Original information is destroyed at Alice's end when the BSM is done. Experimental demonstration of Teleportation of polarized single photon state has been done using standard bi-photonic entangled states [2]. Linear optics, however, does not enable complete BSM.

Van Enk and Hirota proposed [3] a scheme for teleporting superposed coherent states (SCS) using entangled coherent states (ECS). This had the advantage over QT using standard bi-photonic entangled states in that the ECS are more robust against decoherence due to photon absorption than the standard bi-photonic entangled states [4].

Coherent states are the eigenstates of the annihilation operator a, with complex eigenvalue α, given by

$$|\alpha\rangle = e^{-2|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \tag{1}$$

and they have the properties

$$\langle\alpha|\beta\rangle = e^{\alpha^*\beta - \frac{1}{2}(|\alpha|^2 + |\beta|^2)}, \quad \pi^{-1}\int d^2\alpha |\alpha\rangle\langle\alpha| = 1. \tag{2}$$

* prakash˙ranjana1974@rediffmail.com

One may try to write one qubit information by using superposition of states $|\alpha\rangle$ and $|-\alpha\rangle$ in the form

$$|\psi\rangle = \epsilon_+|\alpha\rangle + \epsilon_-|-\alpha\rangle \tag{3}$$

but this is not satisfactory as states $|\alpha\rangle$ and $|-\alpha\rangle$ do not form an orthogonal basis and $\langle\alpha|-\alpha\rangle = e^{-2|\alpha|^2}$. The states may be treated orthogonal only for very large amplitudes α. Orthogonal basis may be obtained by considering superposed coherent states having even or odd number of photons defined by

$$\begin{aligned}|\text{EVEN},\alpha\rangle &= \frac{|\alpha\rangle + |-\alpha\rangle}{\sqrt{2(1+x^2)}} \\ &= \sum_{n=0}^{\infty}\sqrt{\frac{2x}{1+x^2}}\frac{\alpha^{2n}}{\sqrt{2n!}}|2n\rangle, \qquad (4)\\ |\text{ODD},\alpha\rangle &= \frac{|\alpha\rangle - |-\alpha\rangle}{\sqrt{2(1-x^2)}} \\ &= \sum_{n=0}^{\infty}\sqrt{\frac{2x}{1-x^2}}\frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}}|2n+1\rangle, \qquad (5)\end{aligned}$$

where $x \equiv e^{-|\alpha|^2}$. One qubit information may then be written as

$$\begin{aligned}|I\rangle &= \epsilon_+|\alpha\rangle + \epsilon_-|-\alpha\rangle \\ &= A_+|\text{EVEN},\alpha\rangle + A_-|\text{ODD},\alpha\rangle, \qquad (6)\end{aligned}$$

where

$$\begin{aligned}\epsilon_\pm &= \frac{A_+}{\sqrt{2(1+x^2)}} \pm \frac{A_-}{\sqrt{2(1-x^2)}} \\ \text{or } A_\pm &= (\epsilon_+ \pm \epsilon_-)\sqrt{\frac{1\pm x^4}{2}}. \qquad (7)\end{aligned}$$

Normalization condition gives $|A_+|^2 + |A_-|^2 = |\epsilon_+|^2 + |\epsilon_-|^2 + x^2(\epsilon_+\epsilon_- + \epsilon_-\epsilon_+) = 1$. Coefficients $A_\pm$ define the angles (θ,ϕ) on the Bloch sphere by $A_+ = \cos\frac{\theta}{2}, A_- = \sin\frac{\theta}{2}\, e^{i\phi}$.

Van Enk and Hirota [3] suggested use of coherent states for QT. Entangled coherent states are stronger than standard biphotonic entangled states against possible photon transfer to reservoir mode [4]. They took the entangled state of the form

$$|E\rangle_{1,2} = \frac{1}{2\sqrt{1-x^4}}[|\alpha,\alpha\rangle - |-\alpha,-\alpha\rangle], \tag{8}$$

which is maximally entangled. The familiar BSM of the usual QT was replaced by photon counting in two modes. The suggested experimental scheme is shown in FIG. 1.

Explicitly, the scheme shown in FIG. 1, gives the final state for modes 4, 6 and 2 as

$$\begin{aligned}|\psi\rangle_{4,6,2} = &\frac{1}{\sqrt{2(1-x^4)}} \\ &\times\Big[\epsilon_+(|\sqrt{2}\alpha,0\rangle_{4,6}|\alpha\rangle_2 - |0,-\sqrt{2}\alpha\rangle_{4,6}|-\alpha\rangle_2) \\ &+\epsilon_-(|0,\sqrt{2}\alpha\rangle_{4,6}|\alpha\rangle_2 - |-\sqrt{2}\alpha,0\rangle_{4,6}|-\alpha\rangle_2)\Big]. \qquad (9)\end{aligned}$$

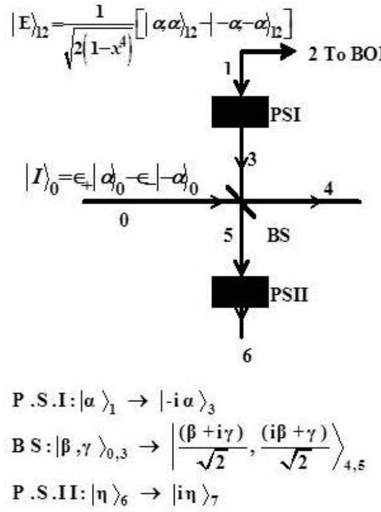

FIG. 1. Shows the scheme of quantum teleportation. Entangled state contains two modes 1 and 2, one of which (mode 2) goes to Bob directly We let Alice pass her part of the entangled state (mode 1) to pass through a phase shifter P.S.I which converts state in mode 1 to state in mode 3. Now Alice mixes state 3 with the state to be teleported (mode 0) by using a 50:50 beam splitter, modifies one of the two outputs (mode 5) by passing it through a phase-shifter P.S.II which changes the state 5 to 6, and then performs photon counting in the two final outputs 4 and 6. This result is then passed to Bob, which helps him to retrieve the information state by performing unitary transformation on the state 2.

van Enk and Hirota separated the coherent states $|\sqrt{2}\alpha\rangle$ in terms of odd and even coherent states and wrote

$$\begin{aligned}|\pm\sqrt{2}\alpha\rangle = &\sqrt{\frac{1}{2}(1+x^4)}|\text{EVEN},\sqrt{2}\alpha\rangle \\ &\pm\sqrt{\frac{1}{2}(1-x^4)}|\text{ODD},\sqrt{2}\alpha\rangle, \qquad (10)\end{aligned}$$

to obtain

$$\begin{aligned}|\psi\rangle_{4,6,2} = &\frac{1}{2}\sqrt{\frac{1+x^2}{(1-x^2)}} \\ &\times\Big[|\text{EVEN},\sqrt{2}\alpha;0\rangle_{4,6}(\epsilon_+|\alpha\rangle_2 - \epsilon_-|-\alpha\rangle_2) \\ &+|0;\text{EVEN},\sqrt{2}\alpha\rangle_{4,6}(-\epsilon_+|-\alpha\rangle_2 + \epsilon_-|\alpha\rangle_2)\Big] \\ &+\frac{1}{2}\Big[|\text{ODD},\sqrt{2}\alpha;0\rangle_{4,6}(\epsilon_+|\alpha\rangle_2 + \epsilon_-|-\alpha\rangle_2) \\ &+|0;\text{ODD},\sqrt{2}\alpha\rangle_{4,6}(-\epsilon_+|-\alpha\rangle_2 + \epsilon_-|\alpha\rangle_2)\Big]. \qquad (11)\end{aligned}$$

This clearly shows that at least one of the count in mode 4 and 6 is always zero. When count in modes (4,6) are (ODD,0) respectively, a replica of information is automatically created at Bob's end. If count are (0,ODD, a unitary transformation changing $|\pm\alpha\rangle \rightarrow |\mp\alpha\rangle$, i.e., reversing the phase of coherent state, generates replica of information at Bob's end. For even counts, however, a replica of $|I\rangle$ may be generated if it was possible to get $|\alpha\rangle \to |\alpha\rangle, |-\alpha\rangle \to -|-\alpha\rangle$ by a unitary transformation. Since such a unitary transformation does not exist, van Enk and Hirota concluded failure in this case and said that success is 1/2.

In our research group, H. Prakash, N. Chandra, Dr. Shivani and I [5] reexamined this scheme and modified the division of photon counting results in two parts, **EVEN and ODD** to three parts: **ZERO, NONZERO EVEN (NZE) and ODD**. The photon counting results are then, **I: (0,0), II: (NZE,0), III: (0,NZE), IV: (ODD,0), V: (0,ODD)**.

Explicitly, separation of $|\sqrt{2}\alpha\rangle$ in terms of vacuum state, nonzero even and odd photon number state is done by writing

$$|\pm\sqrt{2}\alpha\rangle = x|0\rangle + \sqrt{\frac{1-x^2}{2}}|\mathrm{NZE},\sqrt{2}\alpha\rangle \pm\sqrt{\frac{1-x^4}{2}}|\mathrm{ODD},\sqrt{2}\alpha\rangle, \quad (12)$$

where

$$|\mathrm{NZE},\sqrt{2}\alpha\rangle = \frac{|\alpha\rangle + |-\alpha\rangle - 2\sqrt{x}|0\rangle}{\sqrt{2}(1-x)}, x \equiv e^{-|\alpha|^2}. \quad (13)$$

When this is done, we get the state of modes 4, 6 and 2 in the form

$$\begin{aligned}|\psi\rangle_{4,6,2} &= \frac{\sqrt{2}x}{1+x^2}|0\rangle_4|0\rangle_6 A_+|\mathrm{ODD},\alpha\rangle_2 \\ &+\frac{1}{2}\sqrt{\frac{1-x^2}{1+x^2}}\Big\{|\mathrm{NZE},\sqrt{2}\alpha\rangle_4|0\rangle_6\Big[A_+\sqrt{\frac{1-x^2}{1+x^2}}|\mathrm{ODD},\alpha\rangle + A_-\sqrt{\frac{1+x^2}{1-x^2}}|\mathrm{EVEN},\alpha\rangle\Big]_2 \\ &\qquad +|0\rangle_4|\mathrm{NZE},\sqrt{2}\alpha\rangle_6\Big[A_+\sqrt{\frac{1-x^2}{1+x^2}}|\mathrm{ODD},\alpha\rangle - A_-\sqrt{\frac{1+x^2}{1-x^2}}|\mathrm{EVEN},\alpha\rangle\Big]_2\Big\} \\ &+\frac{1}{2}\Big\{|\mathrm{ODD},\sqrt{2}\alpha\rangle_4|0\rangle_6\Big[A_+|\mathrm{EVEN},\alpha\rangle + A_-|\mathrm{ODD},\alpha\rangle\Big]_2 + |0\rangle_4|\mathrm{ODD},\sqrt{2}\alpha\rangle_6\Big[A_+|\mathrm{EVEN},\alpha\rangle - A_-|\mathrm{ODD},\alpha\rangle\Big]_2\Big\}.\end{aligned}$$

Here, the states reaching Bob are seen to depend not always on the information in a complicated way in some cases. Fidelity is also seen at turns to depend on information state in a complicated way. We defined Minimum assured fidelity (MASFI, the minimum of fidelity over all possible information). The conclusions are the same as those of van Enk and Hirota. If both count are zero, Bob gets odd coherent state. Thus F = 1 if $A_+ = 0$ and F = 0 if $A_- = 0$. The MASFI is thus 0 and we conclude a failure. When counts are (NZE,0), in principle, unitary transformation, $U = |\mathrm{EVEN},\alpha\rangle\langle\mathrm{ODD},\alpha| + |\mathrm{ODD},\alpha\rangle\langle\mathrm{EVEN},\alpha|$ gives teleported output of the form

$$\sim B_+|\mathrm{EVEN},\alpha\rangle \pm B_-|\mathrm{ODD},\alpha\rangle \text{ with } B_\pm = A_\pm\sqrt{\frac{1\mp x^2}{1\pm x^2}}. \quad (14)$$

For counts (0,NZE), one more unitary transformation which changes the sign of $|\mathrm{ODD},\alpha\rangle$ but not of $|\mathrm{EVEN},\alpha\rangle$, is required. Since $B_\pm \neq A_\pm$, the fidelity F < 1, but if $|\alpha|^2 >> 1, B_\pm \approx A_\pm$ and F = 1. If the counts are (ODD,0) no unitary transformation is required and Bob gets information directly with F = 1. For counts (0,ODD) however a unitary transformation changing the sign of ODD coherent states is required for QT with F = 1.

Explicitly the teleported states for the various cases of photon counts are

$$\begin{aligned}|T\rangle_I &= |\mathrm{ODD},\alpha\rangle_2 \\ |T\rangle_{II} &= |T\rangle_{III} \\ &\sim A_+\sqrt{\frac{1-x^2}{1+x^2}}|\mathrm{EVEN},\alpha\rangle_2 +A_-\sqrt{\frac{1+x^2}{1-x^2}}|\mathrm{ODD},\alpha\rangle_2, \\ |T\rangle_{IV,V} &= A_+|\mathrm{EVEN},\alpha\rangle_2 + A_-|\mathrm{ODD},\alpha\rangle_2 = |I\rangle_2.\end{aligned}$$

Here $\sim$ denotes unnormalized state. The fidelities are

$$\begin{aligned}F_I &= |A_-|^2 \\ F_{II} = F_{III} &= \frac{1-x^2(|A_+|^2-|A_-|^2)}{1+x^4-2x^2(|A_+|^2-|A_-|^2)} \\ F_{IV} = F_V &= 1.\end{aligned}$$

MASFI is obtained by finding minimum of F against θ and ϕ and its values are

$$\begin{aligned}(\mathrm{MASFI})_I &= 0, \\ (\mathrm{MASFI})_{II,III} &= 1-x^4, \\ (\mathrm{MASFI})_{IV,V} &= 1.\end{aligned}$$

If P_i is probability for occurrence of case i, we define average fidelity

$$F_{av} = \sum_{i=I}^{V} P_i F_i = 1 - \frac{2x^2}{(1+x^2)^2}|A_+|^2[x^2|A_-|^2 + |A_+|^2].$$

This has minimum value, which we call Minimum Average Fidelity (MAVFI), given by

$$(\mathrm{MAVFI})_I = 1 - \frac{2x^2}{(1+x^2)^2} = \frac{1+x^4}{(1+x^2)^2} \approx 1 \text{ for } x << 1.$$

For $|\alpha|^2 = 1$, 2 or 5, MAVFI is seen to be 0.73, 0.9987 and 0.9999 respectively. This leads to, thus, an almost perfect QT. It may be of interest that use of non-maximally entangled resource may improve the fidelity of QT in some cases and Prof. Hari Prakash will speak on it in his talk.

In their classical paper, van Enk and Hirota also considered the problem of decoherence and its effect on QT fidelity. The authors assumed that the coherent photons in the initial state $|\alpha\rangle$ leak to the coupled reservoir vacuum modes and number of photons $|\alpha|^2$ decrease to $\eta|\alpha|^2$. η is called noise parameter. If we consider initial state $|\alpha\rangle$ in mode 0 which is coupled to reservoir mode R0 in initial vacuum state $|0\rangle_{R0}$, the authors assumption is equivalent to assuming

$$|\alpha\rangle_0|0\rangle_{R0} \to |\eta\alpha\rangle_0|k\rangle_{R0} = |\tilde{\alpha}\rangle_0|k\rangle_{R0}, k = \sqrt{1-\eta}\alpha.$$

This was also generalized to multimode reservoir vacuum modes coupled to the signal mode. Thus,

$$|\alpha\rangle_0 \prod_i |0\rangle_{R0i} \to |\eta\alpha\rangle_0 \prod_i |k\rangle_{R0i}, \sum_i |k_i|^2 = (1-\eta)|\alpha|^2.$$

The information state $|I\rangle_0$ coupled with reservoir mode R0 which is initially in vacuum $|0\rangle_{R0}$ therefore undergoes change

$$|I\rangle_0|0\rangle_{R0} \to |\tilde{I}\rangle_{0,R0} \equiv \epsilon_+|\tilde{\alpha}\rangle_0|k\rangle_{R0} + \epsilon_-|-\tilde{\alpha}\rangle_0|-k\rangle_{R0}.$$

Similarly entangled state $|E\rangle_{1,2}$ of modes 1 and 2 coupled to reservoir modes R1 and R2 in initial state $|0\rangle_{R1}$ and $|0\rangle_{R2}$ change to

$$\to \frac{[|\tilde{\alpha}\rangle_1|\tilde{\alpha}\rangle_2|k\rangle_{R1}|k\rangle_{R2}] - |-\tilde{\alpha}\rangle_1|-\tilde{\alpha}\rangle_2|-k\rangle_{R1}|-k\rangle_{R2}]}{\sqrt{2(1-x^4)}}$$

Calculation in the well known way then lead to state of Bobs mode entangled with modes R0, R1 and R2 [6]. Fidelity can be obtained by using $Tr[|I\rangle\langle I|\tilde{\rho_T}]$ where ρ_T is the density operator describing modes 0, 1, 2, R0, R1 and R2.

It may be noted that these results are not identical with the results of van Enk and Hirota as these authors obtained fidelity using $Tr[|\tilde{I}\rangle\langle\tilde{I}|\tilde{\rho_T}]$. We felt that since initial given information is $|I\rangle$, we must compare the final state with $|I\rangle\langle I|$ and find $F = Tr[|I\rangle\langle I|\tilde{\rho_T}]$. .

We may now discuss in brief the results in presence of decoherence. For the (zero, zero) counts case, MASFI is zero only for $\theta = 0$, (i.e., for $A_+ = 1$ or $|I\rangle = |\text{EVEN}\rangle$) and for $\eta = 1$, the noiseless case. For the case with noise, however, MASFI is nonzero in geneal. For count zero and NZE, F is a function of η and θ. For $\eta = 1$, the noiseless F is min at $\theta = \pi/2$. For $1 > \eta > 0.965$, F is min at θ with $0 < \theta < \pi/2$. For $\eta < 0.965$, F is min at $\theta = 0$ and MASFI is therefore F at $\theta = 0$. Its variation against $|\alpha|^2$ is plotted in FIG. 2. For one of counts ODD, for low noise $\eta > 0.721$, MASFI decreses with $|\alpha|^2$. For $\eta < 0.721$, MASFI first increases with $|\alpha|^2$ and then falls as $|\alpha|^2$ is very large. For one count zero

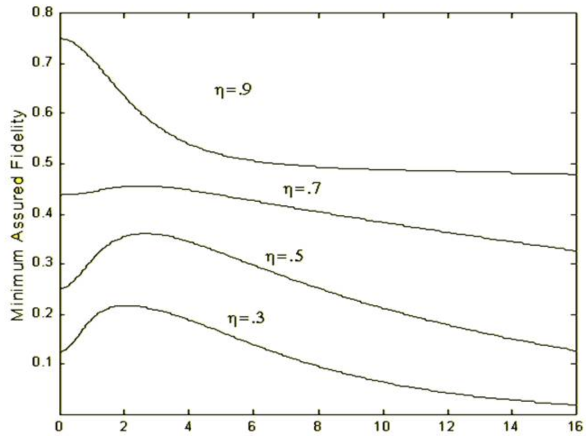

FIG. 2. Variation of MASFI against $|\alpha|^2$ for one of counts NZE.

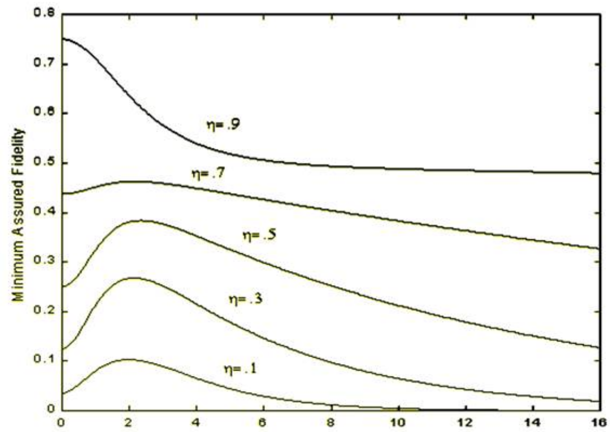

FIG. 3. Variation of MASFI against $|\alpha|^2$ for one of counts ODD.

and the other odd, F is seen to be min at $\theta = \pi$ and the MASFI has variation shown in FIG. 3. For $\eta > 0.738$, MASFI decreases with increase in $|\alpha|^2$. For $\eta < 0.738$, MASFI first increases and then decreases with increase in $|\alpha|^2$. Also it is seen that dependence of F on θ ceases for $|\alpha|^2$ greater than a certain value. For $\eta = 0.9$, this behavior is shown in FIG. 4.

Wang [7] considered a similar problem involving information and entangled states $|I\rangle = \epsilon_+|\alpha,\alpha\rangle + \epsilon_-|-\alpha,-\alpha\rangle$ and $(2(1-x^8))^{-1/2}[|\sqrt{2}\alpha,\alpha,\alpha\rangle - |-\sqrt{2}\alpha,-\alpha,-\alpha\rangle]$. Wang followed the van Enk-Hirota scheme and reported success 1/2. We modified [9] this scheme also and got fidelity very close to 1. If we write information as

$$\begin{aligned} |I\rangle &= \epsilon_+|\alpha,\alpha\rangle + \epsilon_-|-\alpha,-\alpha\rangle \\ &= A_+|\text{EVEN};\alpha,\alpha\rangle + A_-|\text{ODD};\alpha,\alpha\rangle, \end{aligned} \tag{15}$$

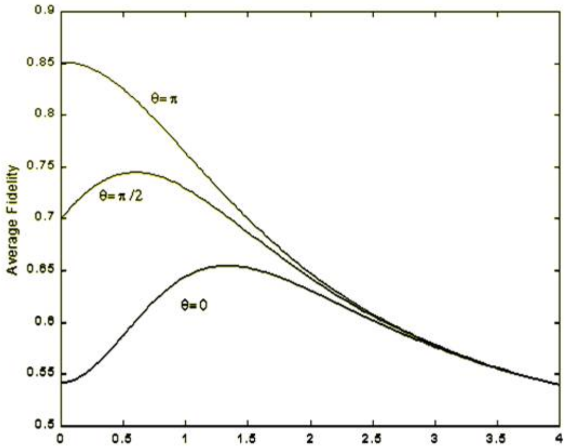

FIG. 4. Variation of average fidelity with $|\alpha|^2$ for some value of θ for $\eta = 0.9$.

where

$$|\text{EVEN};\alpha,\alpha\rangle = \frac{1}{\sqrt{2(1+x^4)}}[|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle],$$
$$|\text{ODD};\alpha,\alpha\rangle = \frac{1}{\sqrt{2(1-x^4)}}[|\alpha,\alpha\rangle - |-\alpha,-\alpha\rangle],$$

and coefficients $\epsilon_\pm$ and $A_\pm$ are inter-related. The experimental set up is shown in FIG. 5.

Photon counting is done in the modes 11 and 12 and one of the counts is always zero. The possible cases of counts are I: (zero, zero), II: (nonzero even, zero), III: (zero, nonzero-even), IV: (odd, zero) and V: (zero, odd). The unitary transformations are seen to be

$$\begin{aligned} U_I &= U_{IV} = 1, \\ U_{II,III} &= |\text{EVEN};\alpha,\alpha\rangle\langle\text{ODD};\alpha,\alpha| \\ &\quad \pm|\text{ODD};\alpha,\alpha\rangle\langle\text{EVEN};\alpha,\alpha|, \\ U_V &= |\text{EVEN};\alpha,\alpha\rangle\langle\text{EVEN};\alpha,\alpha| \\ &\quad -|\text{ODD};\alpha,\alpha\rangle\langle\text{ODD};\alpha,\alpha|. \end{aligned}$$

Giving the teleported states

$$\begin{aligned} |T\rangle_I &= |\text{ODD};\alpha,\alpha\rangle, \\ |T\rangle_{II} &= |T\rangle_{III} \sim A_+\sqrt{\frac{1-x^4}{1+x^4}}|\text{EVEN};\alpha,\alpha\rangle \\ &\quad +A_-\sqrt{\frac{1+x^4}{1-x^4}}|\text{ODD};\alpha,\alpha\rangle, \\ |T\rangle_{IV} &= |T\rangle_V = |I\rangle, \end{aligned}$$

and the fidelities

$$F_I = |A_-|^2, \quad F_{IV} = F_V = 1,$$
$$F_{II} = F_{III} = 1 - \frac{1 - x^4(|A_+|^2 - |A_-|^2)}{1 + x^8 - 2x^4(|A_+|^2 - |A_-|^2)}.$$

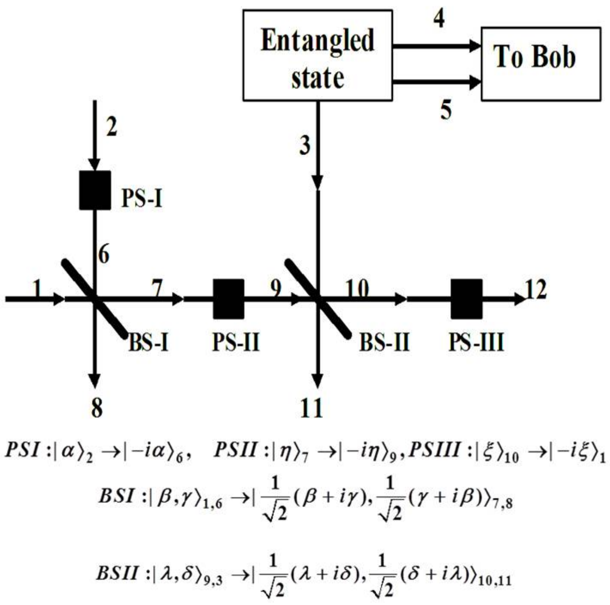

FIG. 5. Numerals 1, 2, ..., 6 refers to modes. Entangled states of modes 1 and 2 are to be teleported to bob. Out of , state in mode 3 goes to Alice while states in modes 4 and 5 go to Bob. Alice (i) converts state 2 to state 6 by using phase shifter PS-I, (ii) mixes state 6 with state 1 using a beam splitter BS-I, (iii) modifies output in 7 to state 9 using phase shifter PS-II, (iv) mixes state 9 with state 3 using beam splitter BS-II (v) modifies output in 10 to state 12 using phase shifter PS-III, and (vi) performs photon counting in mode 11 and 12. The results of photon counting, conveyed to Bob by a classical channel helps him construct the entangled state by making unitary transformation on state of mode 4 and 5

This leads to average fidelity

$$F_{av} = 1 - \frac{2x^4|A_+|^2(x^4|A_-|^2 + |A_+|^2)}{(1+x^4)^2}.$$

Minimum of this is the Minimum Average Fidelity (MAVFI),

$$\text{MAVFI} = 1 - \frac{2x^4}{(1+x^4)^2} \text{ for } A_- = 0.$$

To estimate, we see that for $|\alpha|^2 \approx 2, \text{MAVFI} = 1 - 0.98 \times 10^{-4}$. The QST is thus almost perfect.

We considered the effect of decoherence also [8] and obtained recently similar to those discussed in previous case. The MASFI is zero only if $\eta = 1$ and inforamtion is even coherent state. For low noise MASFI decreases with increase in $|\alpha|^2$ but at higher noise, it first increases and then decreases on increasing $|\alpha|^2$. The average fidelity is independent of information for appreciable $|\alpha|^2$.

We also consider [9] controlled QT using 4-partite states involving, in addition to sender Alice and receiver Bob, two extra players Clair and David. As a matter of fact, we re-examined work of N. B. Ann [10] who reported success 1/4 in limit $\alpha \to \infty$. We found almost perfect

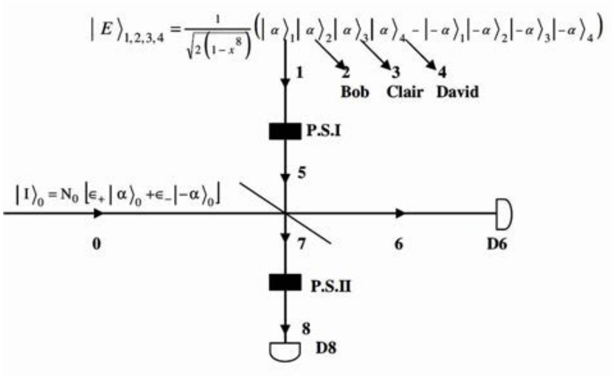

FIG. 6. Controlled quantum teleportation using four-particle state.

success for appreciable $|\alpha|$. The experimental procedure is as explained in FIG. 6. Llight beam of mode 1 falls on phase shifter I which changes state $|\alpha\rangle$ to $|i\alpha\rangle$ and this beam and light beam of information mode are incident on a symmetric beam splitter. One output passes through a phase shifter II which changes state $|\alpha\rangle$ to $|i\alpha\rangle$.

Photon counting is done in the other output (mode 6) and the phase shifted output (mode 8). Clair and David also count photons in their modes 3 and 4. The photon counts in modes 6 and 8 and sum of these two counts in modes 3 and 4 decide the unitary transformation which is done by Bob.

Proceeding exactly in the same as explained earlier, the minimim average fidelity is seen to be $1-[2x^2/(1+x^2)^2]$. For $|\alpha| = 5$, this is 0.9999. We considered the effect of decoherence also [11]. Conclusions similar to those in previous two cases are obtained.

We now study entanglement diversion between two pairs of entangled coherent sates. This problem involves three remote parties Alice, Bob and Clair. Alice and Bob are connected to each other by sharing an entangled state $|\phi\rangle_{1,3}$ of mode 1 with Alice and mode 3 with Bob. Similarly Alice and Clair are connected to each other by sharing an entangled state $|\psi\rangle_{2,4}$ of mode 2 with Alice and mode 4 with Clair. Connected persons can do QT between them. But as Bob's mode 3 is not entangled with Clair's mode 4, no QT is possible between Bob and Clair. Xin-Hua [12] gave a scheme for entanglement diversion which makes Bob's and Clair's mode entangled. Xin-Hua reported success of 1/2. We modify this scheme and obtain near perfect entanglement diversion [13].

Photon counting is done in (see FIG. 7) modes 6 and 8 and if $|\phi\rangle_{1,3}$ and $|\psi\rangle_{2,4}$ both and $(2(1-x^4))^{-1/2}[|\alpha,\alpha\rangle - |-\alpha,-\alpha\rangle]$ the state of modes 6, 8, 3 and 4 is seen to be, i.e., $|\psi\rangle_{6,8,3,4}$ is equal to

$$\frac{1}{2(1-x^4)}\Big[\Big\{x|0\rangle_6|0\rangle_8(|\alpha\rangle - |-\alpha\rangle)_3(|\alpha\rangle - |-\alpha\rangle)_4\Big\}$$
$$+\frac{1-x^2}{\sqrt{2}}\Big\{|\mathrm{NZE},\sqrt{2}\alpha\rangle_6|0\rangle_8(|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle)_{3,4} - |0\rangle_6|\mathrm{NZE},\sqrt{2}\alpha\rangle_8(|\alpha,-\alpha\rangle + |-\alpha,\alpha\rangle)_{3,4}\Big\}$$
$$+\sqrt{\frac{1-x^4}{2}}\Big\{|\mathrm{ODD},\sqrt{2}\alpha\rangle_6|0\rangle_8(|\alpha,\alpha\rangle - |-\alpha,-\alpha\rangle)_{3,4} - |0\rangle_6|\mathrm{ODD},\sqrt{2}\alpha\rangle_8(|\alpha,-\alpha\rangle - |-\alpha,\alpha\rangle)_{3,4}\Big\}\Big].$$

If Alice performs photon counting on her modes 6 and 8 and conveys result to either of Bob and Clair, the receiver can perform a unitary transformation and try to make the modes 3 and 4 perfectly entangled.

Alice's results of photon counts in modes 6 and 8 can be I: (0,0), II: (NZE,0), III: (0,NZE), IV: (ODD,0), V: (0,ODD). For case I entanglement is zero and F = 0. But, probability P_I for occurrence of this case is $x^2/(1+x^2)^2$, and its plot is shown in above of the FIG. 8. It can be seen that $P_I \approx 0$ for $|\alpha|^2 = 3$. Plots of $P_{II,III}$ and the fidelity are shown in FIG. 9.

For cases IV and V the probability of occurrence is 1/4 and the Fidelity is 1. Variation of average fidelity with $|\alpha|^2$ is shown in right of the FIG. 9.

We studied the effect of decoherence on this scheme of entanglement diversion also [14]. Variation of Fidelity with $|\alpha|^2$ for various values of η for cases (II, III) and (IV, V) are shown in the FIG. 10.

We studied a similar phenomenon of swapping between two pairs of non-orthogonal entangled coherent states [15] and the effect of noise on it [16].

[1] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).

[2] D. Boumweester, J-W. Pan, K. Mattle, M. Eibl. H. einfurter and A. Zeilinger, Nature **390**, 575 (1997); D. Boschi, S. Branca, F. De Martini, L. Hardy and S. Popescu, Phys. Rev. Lett. **80**, 1121 (1998); Y-H. Kim, S.P. Kulik and Y. Shilo, Phys. Rev. **86**, 1370 (2001);

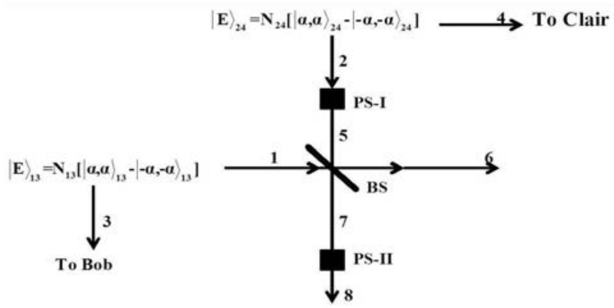

FIG. 7. Proposed experimental setup for entanglement diversion. Numerals 1, 2, ..., 8 refers to modes. States in modes 1 and 2 are with Alice while state 3 and 4 are with Bob and Clair respectively. Alice (i) converts state 2 to state 5 using phase shifter PS-I, (ii) mixes state 1 with state 5 using a beam splitter BS, (iii) modifies output in 7 to state 8 using phase shifter PS-II, and (iv) performs photon counting in modes 6 and 8. The result, conveyed to Bob helps him to perform a unitary transformation on state 3 and 4 and generate an exact replica of entangled state.

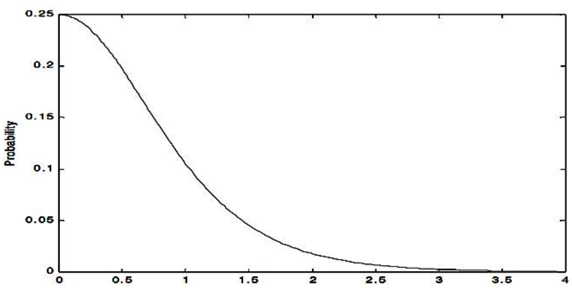

FIG. 8. Variation of P_I with $|\alpha|^2$.

I. Marcikie, H de Reidmatten, W. Vittell, H. Zbinden and N. Gisin, Nature **421**, 509 (2003); R. Ursin, T. Jennewein, M. Aspelmeyer, R. Kaltenbaek, M. Lindenthal, P. Walther and A. Zeilinger, Nature **430**, 849 (2004), X-M. Jin, J-G. Ren, B. Yang, Z-H. Yi, F. Zhou, X-F. Xu, S-K. Wang, D. Yang, Y-F. Hu, S. Jiang, T. Yang, H. Yin, K. Chen, C-Z Peng and J-W Pan, Nature Photonics **4**, 376 (2010); J. Yin, H. Lu, J-G Ren, Y. Cao, H-L.Yong, Y-P.Wu, C. Liu, S-K. Liao, Y. Jiang, X-D.Cai, P.Xu, G-S. Pan, J-Y. Wsang, Y-A. Chen, C-Z. Peng and J-W. Pan, arXiv: quant-ph/1205.2024v1.

[3] S.J. van Enk and O. Hirota, Phys. Rev. A **64**, 022313 (2001).

[4] O.Hirota, S.J. van Enk, K. Nakamura, M, sohma and K. Kato, arXiv/quant-ph/0101096.

[5] H.Prakash, N. Chandra, R. Prakash and Shivani, Phys. Rev. A **75**, 044305 (2007), also published in Vir. J. Quantum Inf., Vol. **7**, Issue 5, May (2007).

[6] H.Prakash, N. Chandra, R. Prakash and Shivani, J. Phys. B: At. Mol. Opt. Phys. **40**, 1613 (2007).

[7] X. Wang, Phys. Rev. A **64**, 022302 (2001).

[8] H. Prakash , N. Chandra, R. Prakash and Shivani, Int. J. Quantum Info. B **6**, 1077 (2008).

[9] H. Prakash , N. Chandra, R. Prakash and Shivani, Int. J. Mod. Phys. B **24**, 3383 (2010).

[10] N. Ba Ann, Phys. Rev. A **68**, 022321 (2003).

[11] Shivani A. Kumar, H. Prakash , N. Chandra, and R. Prakash, J. Quantum Info. Sc. **2**, 123 (2012).

[12] C. Xin-Hua, G. Jie-Rong, N.Jian-Jun and J. Jin-Ping, Chin. Phys. **15**, 0488 (2006)

[13] H. Prakash, N. Chandra, R. Prakash and Shivani, Int. J. Mod. Phys. B **24**, 3331(2010).

[14] H. Prakash, N. Chandra, R. Prakash and Shivani, Int. J. Mod. Phys. B **23**, 585 (2009).

[15] H. Prakash, N. Chandra, R. Prakash and Shivani, Int. J. Mod. Phys. B **23**, 2083 (2009).

[16] H. Prakash, N. Chandra, R. Prakash and Shivani, Mod. Phys. Lett. B **27**, 1350017 (2013).

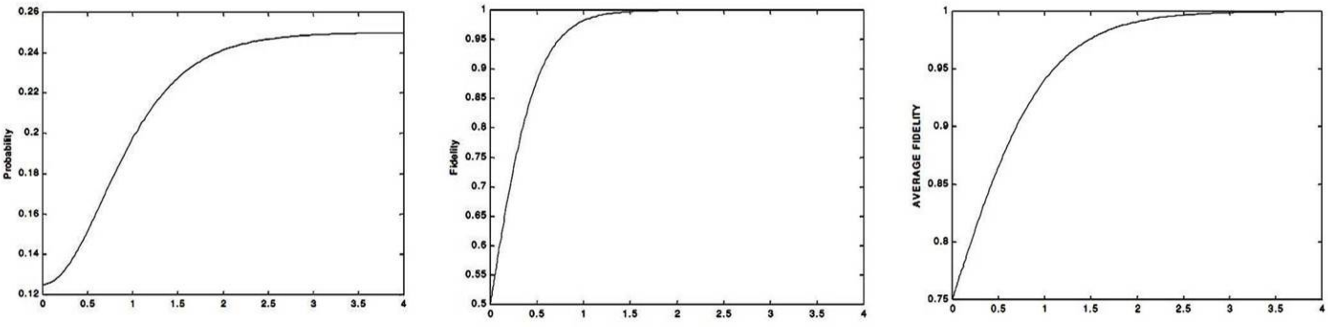

FIG. 9. Variation of $P_{II} = P_{III}$ and of fidelity with $|\alpha|^2$ and for cases IV & V variation of Average Fideltiy with $|\alpha|^2$.

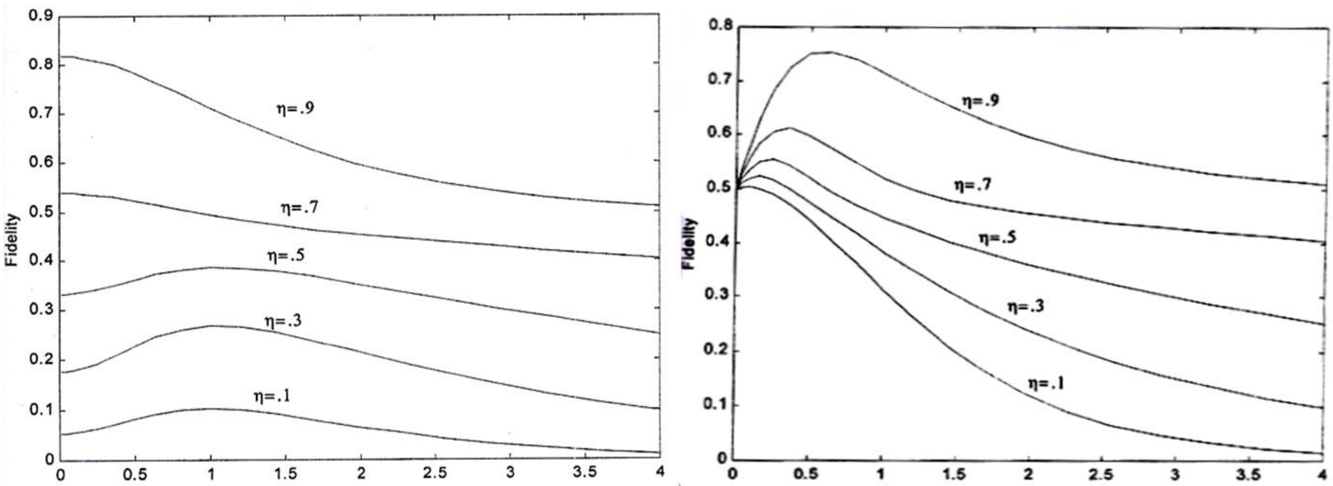

FIG. 10. Variation of Fidelities for cases II & III and for cases of IV & V with $|\alpha|^2$ for various values of η.