Entanglement-assisted classical communication using quasi-Bell states

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In entanglement-assisted classical communication, to study influence of noise in quantum channel is important. Entangled states by nonorthogonal states called "quasi-Bell states", such as coherent states of light, have been shown to be capable of "perfect entanglement" and are expected to be robust against attenuation in quantum channel. Recently, a property of a capacity of a quantum channel assisted by a degraded quasi-Bell state was shown by using an approximately executable encoding. In the present paper, we consider strictly executable encodings and show the optimum encoding and properties of capacity.

I. INTRODUCTION

Entanglement is known as an important resource in quantum information systems. In application, it is desirable to use complete entanglement or a maximally entangled state. Entangled states by nonorthogonal states called "quasi-Bell states [1]", such as coherent states of light, have been shown to be capable of "perfect entanglement" and are expected to be robust against attenuation in quantum channel. Since then, the quasi-Bell states have been received much attention and a lot of papers have been published (e.g. [3–7]). We consider an application of quasi-Bell states to the entanglement-assisted classical communication [8], which is one of two major application protocols of entanglement as quantum teleportation. An entanglement-assisted classical communication or a quantum superdense coding is a protocol of classical information transmission using entanglement, that was proposed by Bennett and Wiesner [8].

A classical information transmission assisted by a quasi-Bell state was considered by Hirota et al. [9]. In [9], they assumed an ideal channel and remarked that an investigation of an effect of loss is desired. Recently, we considered two kinds of effects of loss on the entanglement-assisted classical communication by a quasi-Bell state [10, 11]. One correspoonds that a quasi-Bell state is attenuated in its sharing process (before performing the protocol) and the other corresponds that a lossy channel is used in the entanglement-assisted classical communication protocol. However, as explained in Section 4.2 in [11], an approximate encoding was assumed in all these papers. The approximation works when amplitude of coherent states are sufficiently large.

In the present paper, we use rigorously realizable encodings by introducing an orthonormal basis in the Hilbert space spanned by the coherent-states which are components of the quasi-Bell state. Then we show properties of an entanglement-assisted classical capacity.

II. QUASI-BELL STATES

Quasi-Bell states are entangled qubit states based on a nonorthogonal computational basis [1]. In general, arbitrary two (nonorthogonal) pure-states generate four quasi-Bell states. In contrast, four Bell states are generated by two orthogonal pure-states. Since the dimension of the space spanned by the two nonorthogonal states is $d = 2$, the degree of entanglement of the quasi-Bell states is at most $\log_2 d = \log_2 2 = 1$[ebit]. It was shown that the degree of entanglement of two of the four quasi-Bell states attains the upper limit 1[ebit] and was said that the two of the four quasi-Bell states have "perfect entanglement [1]".

Using coherent states of light as nonorthogonal states, they are called entangled coherent states [3, 12] which is attractive as the third type of entangled states (cf. Bell states and two-mode squeezed states). In the present paper, we consider entangled coherent states.

A coherent state whose complex amplitude is α is expressed as $|\alpha\rangle$. Let $|0_{\rm L}\rangle = |0\rangle$ and $|1_{\rm L}\rangle = |\alpha\rangle$ be the (nonorthogonal) logical qubit states. Then quasi-Bell states by coherent-states are expressed as

$$|\Psi_1\rangle_{\rm AB} = h_1(|0\rangle_{\rm A}|\beta\rangle_{\rm B} + |\alpha\rangle_{\rm A}|0\rangle_{\rm B}), \quad (1)$$

$$|\Psi_2\rangle_{\rm AB} = h_2(|0\rangle_{\rm A}|\beta\rangle_{\rm B} - |\alpha\rangle_{\rm A}|0\rangle_{\rm B}), \quad (2)$$

$$|\Psi_3\rangle_{\rm AB} = h_3(|0\rangle_{\rm A}|0\rangle_{\rm B} + |\alpha\rangle_{\rm A}|\beta\rangle_{\rm B}), \quad (3)$$

$$|\Psi_4\rangle_{\rm AB} = h_4(|0\rangle_{\rm A}|0\rangle_{\rm B} - |\alpha\rangle_{\rm A}|\beta\rangle_{\rm B}), \quad (4)$$

where

$$h_1 = h_3 = 1/\sqrt{2(1+\kappa_{\rm A}\kappa_{\rm B})},$$
$$h_2 = h_4 = 1/\sqrt{2(1-\kappa_{\rm A}\kappa_{\rm B})},$$
$$\kappa_{\rm A} = \langle 0|\alpha\rangle = \langle \alpha|0\rangle = \exp(-2|\alpha|^2),$$
$$\kappa_{\rm B} = \langle 0|\beta\rangle = \langle \beta|0\rangle = \exp(-2|\beta|^2),$$

and α and β are assumed to be non-negative real numbers. When $\alpha = \beta$, it was shown that $|\Psi_2\rangle$ and $|\Psi_4\rangle$ have perfect entanglement.

III. QUANTUM SUPERDENSE CODING

It had been shown that the use of entanglement enhances classical communications [8]. The protocol is called a quantum superdense coding or an entanglement-assisted classical communication. As an example, in a qubit system, maximum bits obtained at the receiver

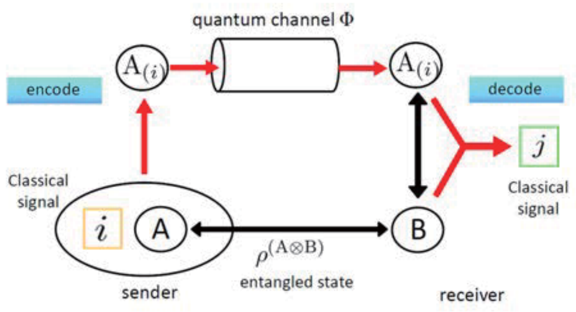

FIG. 1. Schematic diagram of a classical communication assisted by an entangled state.

without initial shared entanglement is only 1[bit] by transmitting 1-qubit, whereas 2[bits] can be obtained by entanglement assistance. The quantum superdense coding consists of the following steps:

1. Alice (sender) and Bob (receiver) share an entangled state $\hat{\rho}^{(\mathrm{A}\otimes\mathrm{B})}$ beforehand.
2. Alice encodes a classical signal i by applying an encoding function $\mathcal{E}_i^{(\mathrm{A})}$ to her side of $\hat{\rho}^{(\mathrm{A}\otimes\mathrm{B})}$.
3. Bob obtains the states $\hat{\bar{\rho}}_i^{(\mathrm{A}\otimes\mathrm{B})}$ by receiving Alice's part of $\hat{\rho}_i^{(\mathrm{A}\otimes\mathrm{B})}$ through a quantum channel Φ.
4. Performing the quantum combined measurement on Bob's side, he obtains a classical signal j.

A. Encoding

An encoding function $\mathcal{E}_i^{(\mathrm{A})}$ is local operation for the mode A of $\hat{\rho}^{(\mathrm{A}\otimes\mathrm{B})}$ corresponding to a classical signal i. Since the mode B is unchanged in the encoding process, the local operation for the mode B is the identity operator $I^{(\mathrm{B})}$. Thus, the encoding is represented as follows:

$$\hat{\rho}_i^{(\mathrm{A}\otimes\mathrm{B})} = \left[\mathcal{E}_i^{(\mathrm{A})} \otimes I^{(\mathrm{B})}\right] \hat{\rho}^{(\mathrm{A}\otimes\mathrm{B})} \left[\mathcal{E}_i^{(\mathrm{A})} \otimes I^{(\mathrm{B})}\right]^{\dagger}. \quad (5)$$

B. Classical capacity assisted by entanglement

For a quantum channel Φ, the capacity (Holevo capacity) of fixed an encoding function $\mathcal{E}_i^{(\mathrm{A})}$ and an entangled state $\hat{\rho}^{(\mathrm{A}\otimes\mathrm{B})}$ is

$$C_{\mathrm{ea}}^{(1)}(\mathcal{E}_i^{(\mathrm{A})}, \hat{\rho}^{(\mathrm{A}\otimes\mathrm{B})}, \Phi) = \max_{\{\xi_i\}} \left\{ S\Big(\sum_i \xi_i \hat{\bar{\rho}}_i^{(\mathrm{A}\otimes\mathrm{B})}\Big) - \sum_i \xi_i S\big(\hat{\bar{\rho}}_i^{(\mathrm{A}\otimes\mathrm{B})}\big) \right\}, \quad (6)$$

where $S(\rho) = -\mathrm{Tr}\rho\log\rho$ and $\{\xi_i\}$ is an *a priori* distribution assigned to classical signals $\{i\}$. Note that the capacity $C_{\mathrm{ea}}^{(1)}(\Phi)$ is defined as the maximum value of the capacity (6) with respect to shared entangled states and encodings. In the same way, the capacity $C_{\mathrm{ea}}^{(n)}(\Phi)$ is defined for the channel $\Phi^{\otimes n}$ and the limit

$$C_{\mathrm{ea}} = \lim_{n\to\infty} \frac{1}{n} C_{\mathrm{ea}}^{(n)}(\Phi), \quad (7)$$

is so called the entanglement-assisted classical capacity [13, 14]. In the present paper, however, we compute the one-shot capacity (6) when a quasi-Bell state is shared.

IV. ATTENUATION ON QUASI-BELL STATES

A. Attenuation in sharing process

Alice prepares the following entangled state:

$$|\Psi_4\rangle = h_4(|0\rangle_{\mathrm{A}}|0\rangle_{\mathrm{B}} - |\alpha\rangle_{\mathrm{A}}|\beta\rangle_{\mathrm{B}}). \quad (8)$$

We assume that the mode B of the $|\Psi_4\rangle$ is attenuated when Alice and Bob share the entangled state. Degradation of entanglement by an attenuated quantum channel can be represented as an interaction between the mode B and an external vacuum mode or an environment mode E. Let η_{B} be the transmissivity $(0 \le \eta_{\mathrm{B}} \le 1)$ of the attenuated channel. The interaction of the mode B and the external mode E is represented as

$$U_{\mathrm{BE}}|\beta\rangle_{\mathrm{B}}|0\rangle_{\mathrm{E}} = |\sqrt{\eta_{\mathrm{B}}}\beta\rangle_{\mathrm{B}}|\sqrt{1-\eta_{\mathrm{B}}}\beta\rangle_{\mathrm{E}}. \quad (9)$$

Thus the output state for the compound system of A, B, and E is

$$\begin{aligned} |\Psi_4\rangle_{\mathrm{ABE}} &= (I_{\mathrm{A}} \otimes U_{\mathrm{BE}})(|\Psi_4\rangle_{\mathrm{AB}} \otimes |0\rangle_{\mathrm{E}}) \\ &= \tilde{h}_4\big(|0\rangle_{\mathrm{A}}|0\rangle_{\mathrm{B}}|0\rangle_{\mathrm{E}} - |\alpha\rangle_{\mathrm{A}}|\sqrt{\eta_{\mathrm{B}}}\beta\rangle_{\mathrm{B}}|\sqrt{1-\eta_{\mathrm{B}}}\beta\rangle_{\mathrm{E}}\big), \quad (10) \end{aligned}$$

where

$$\tilde{h}_4 = \frac{1}{\sqrt{2(1 - L_{\mathrm{B}}\kappa_{\mathrm{A}}\kappa'_{\mathrm{B}})}}, \quad (11)$$

$$\kappa_{\mathrm{A}} = \langle 0|\alpha\rangle, \quad (12)$$

$$\kappa'_{\mathrm{B}} = \langle 0|\sqrt{\eta_{\mathrm{B}}}\beta\rangle, \quad (13)$$

$$L_{\mathrm{B}} = \langle 0|\sqrt{1-\eta_{\mathrm{B}}}\beta\rangle. \quad (14)$$

As a result, the density operator $\hat{\rho}^{(\mathrm{A}\otimes\mathrm{B})}$ for mode A and B after attenuation is

$$\begin{aligned} \hat{\rho}^{(\mathrm{A}\otimes\mathrm{B})} &= \mathrm{Tr}_{\mathrm{E}}(|\Psi_4\rangle_{\mathrm{ABE}}\langle\Psi_4|) \\ &= (\tilde{h}_4)^2\Big\{|0\rangle_{\mathrm{A}}|0\rangle_{\mathrm{B}}\langle 0|_{\mathrm{A}}\langle 0| + |\alpha\rangle_{\mathrm{A}}|\sqrt{\eta_{\mathrm{B}}}\beta\rangle_{\mathrm{B}}\langle\sqrt{\eta_{\mathrm{B}}}\beta|_{\mathrm{A}}\langle\alpha| \\ &\quad -L_{\mathrm{B}}\big(|0\rangle_{\mathrm{A}}|0\rangle_{\mathrm{B}}\langle\sqrt{\eta_{\mathrm{B}}}\beta|_{\mathrm{A}}\langle\alpha| + |\alpha\rangle_{\mathrm{A}}|\sqrt{\eta_{\mathrm{B}}}\beta\rangle_{\mathrm{B}}\langle 0|_{\mathrm{A}}\langle 0|\big)\Big\}. \quad (15) \end{aligned}$$

B. Encoding

In this study, we consider rigorously realizable encodings. Here, we mean by a *rigorously realizable encoding* that the encoding is unitary. For this purpose, we introduce an orthonormal basis $\{|\omega_0\rangle_A, |\omega_1\rangle_A\}$ and consider the matrix representation of a quasi-Bell state by the basis. How to select the basis ? The first condition is that the logical qubit states $|0_L\rangle_A = |0\rangle_A$ and $|1_L\rangle_A = |\alpha\rangle_A$ tend to the basis vectors $|\omega_0\rangle_A$ and $|\omega_1\rangle_A$ in the limit of infinite amplitude:

$$|0\rangle_A \to |\omega_0\rangle_A,\ |\alpha\rangle_A \to |\omega_1\rangle_A\ (\alpha \to \infty) \tag{16}$$

The second condition is the basis is sufficiently close to the logical qubits. As a result, the measurement states of the square-root measurement (SRM) [15] for binary quantum signals $\{|0\rangle_A, |\alpha\rangle_A\}$ is suitable as the basis. The SRM is well known as an optimum or asymptotically optimum measurement for quantum signals (e.g. [15–19]). Moreover, for pure-state signals, it is often called the least square measurement (LSM) since the minimum distance between the signals and the measurement states is attained by the SRM [19]. Therefore, $\{|\omega_0\rangle_A, |\omega_1\rangle_A\}$ are the measurement states of the optimum quantum measurement for the signals $\{|0\rangle_A, |\alpha\rangle_A\}$ and are represented as [20, 21]

$$|\omega_0\rangle_A = \sqrt{\varepsilon_{A+}}|0\rangle_A - \sqrt{\varepsilon_{A-}}|\alpha\rangle_A, \tag{17}$$
$$|\omega_1\rangle_A = \sqrt{\varepsilon_{A-}}|0\rangle_A - \sqrt{\varepsilon_{A+}}|\alpha\rangle_A, \tag{18}$$

where

$$\varepsilon_{A\pm} = \frac{1 \pm \sqrt{1-\kappa_A^2}}{2(1-\kappa_A^2)}. \tag{19}$$

We can describe $|0\rangle_A$ and $|\alpha\rangle_A$ by using $|\omega_0\rangle_A$ and $|\omega_1\rangle_A$ as

$$\begin{aligned}|0\rangle_A &= \frac{1}{\varepsilon_{A+}-\varepsilon_{A-}}(\sqrt{\varepsilon_{A+}}|\omega_0\rangle_A - \sqrt{\varepsilon_{A-}}|\omega_1\rangle_A)\\ &= \frac{1}{\varepsilon_{A+}-\varepsilon_{A-}}\begin{bmatrix}\sqrt{\varepsilon_{A+}}\\ -\sqrt{\varepsilon_{A-}}\end{bmatrix},\end{aligned} \tag{20}$$

$$\begin{aligned}|\alpha\rangle_A &= \frac{1}{\varepsilon_{A+}-\varepsilon_{A-}}(\sqrt{\varepsilon_{A-}}|\omega_0\rangle_A - \sqrt{\varepsilon_{A+}}|\omega_1\rangle_A)\\ &= \frac{1}{\varepsilon_{A+}-\varepsilon_{A-}}\begin{bmatrix}\sqrt{\varepsilon_{A-}}\\ -\sqrt{\varepsilon_{A+}}\end{bmatrix}.\end{aligned} \tag{21}$$

Similarly, we can describe qubit states for the mode B.

Since we give a matrix representation of the quisi-Bell state, it is sufficient to represent encodings by unitary matrices. We consider the following unitary matrices which correspond to the local operation $\mathcal{E}_i^{(A)}$ for the mode A.

$$U_A^{(i)} = \tilde{R}(\theta^{(i)}) = \begin{cases} R_1(\theta^{(i)}) & (0 \le \theta^{(i)} < 2\pi),\\ R_2(\theta^{(i)}) & (2\pi \le \theta^{(i)} < 4\pi),\end{cases} \tag{22}$$

where

$$R_1(\theta^{(i)}) = \begin{bmatrix}\cos(\theta^{(i)}) & -\sin(\theta^{(i)})\\ \sin(\theta^{(i)}) & \cos(\theta^{(i)})\end{bmatrix}, \tag{23}$$

$$R_2(\theta^{(i)}) = \begin{bmatrix}\cos(\theta^{(i)}) & \sin(\theta^{(i)})\\ \sin(\theta^{(i)}) & -\cos(\theta^{(i)})\end{bmatrix}, \tag{24}$$

and I_B is the identity map for the Bob's system

$$I_B = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}. \tag{25}$$

C. Attenuation when the protocol is performed

We consider the second loss which corresponds to an attenuation when the protocol is performed. In performing the protocol, Alice transmits her side of $\hat{\rho}_i^{(A\otimes B)}$ through an attenuated quantum channel whose transmissivity is $\eta_A (0 \le \eta_A \le 1)$. Mathematical treatment of the attenuation is the same as that described in Section IV.A.

V. EFFECT OF LOSS ON A QUASI-BELL STATE

A. Determining encoding

Before considering property of capacity, we determine the *fixed* encoding function. We use the optimum encoding when the *a priori* probability distribution is uniform. We numerically derive the optimum encoding that maximizes the Holevo information. As a result, the encoding maximizes the Holevo information when $\theta^{(1)} = 0, \theta^{(2)} = \frac{\pi}{2}, \theta^{(3)} = 2\pi, \theta^{(4)} = \frac{5}{2}\pi$. This means the unitary matrices corresponding to the optimum encoding are

$$U_A^{(1)} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = I, \tag{26}$$

$$U_A^{(2)} = \begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} = Y, \tag{27}$$

$$U_A^{(3)} = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} = Z, \tag{28}$$

$$U_A^{(4)} = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} = X. \tag{29}$$

Here, I, X, Y, and Z are the identity, the bit flipping, the bit phase flipping, and the phase flipping operators, respectively. Thus, we find that the well known I, X, Z, and Y correspond to the optimum encoding if we use the orthonormal basis $\{|\omega_0\rangle_A, |\omega_0\rangle_B\}$ for the matrix representation.

Using the above encoding, we compute the capacity. Here, maximization with respect to an *a priori* distribution $\{\xi_i\}$ is performed numerically. It turns out that the capacity is attained by the uniform distribution.

B. Property of capacity

In this section, we consider property of the entanglement-assisted classical capacity through an attenuated quantum channel when a quasi-Bell state which

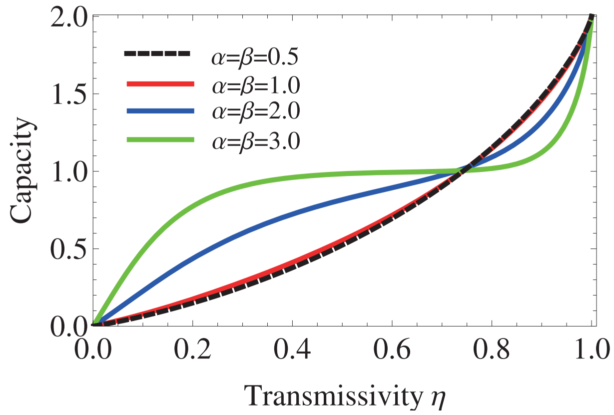

FIG. 2. Classical capacities assisted by quasi-Bell states with respect to the transmissivity $\eta_A = \eta_B = \eta$ of the attenuated quantum channel. The black dashed line, the red solid line, the blue solid line, and the green solid line correspond to the cases that coherent amplitudes are $\alpha = \beta = 0.5, 1.0, 2.0,$ and 3.0, respectively.

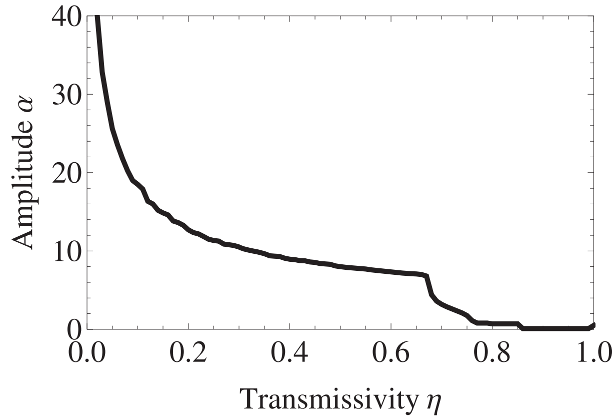

FIG. 3. Optimized coherent amplitude α with respect to the transmissivity $\eta_A = \eta_B = \eta$ of the attenuated quantum channel.

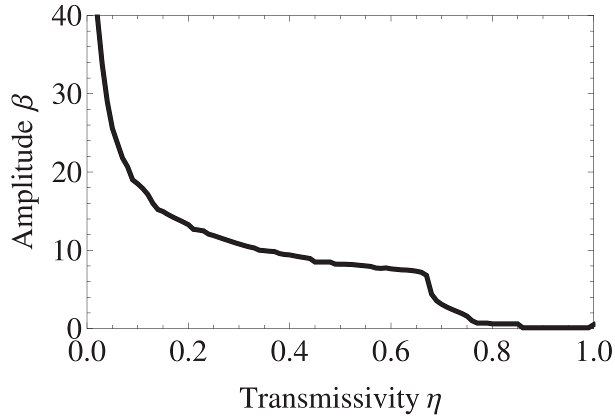

FIG. 4. Optimized coherent amplitude β with respect to the transmissivity $\eta_A = \eta_B = \eta$ of the attenuated quantum channel.

is already attenuated in its sharing process is used as shared initial entanglement. That is, both losses exist. Here, we assume $\eta_A = \eta_B = \eta$. Figure 2 shows the classical capacities assisted by quasi-Bell states with respect to the transmissivity $\eta_A = \eta_B = \eta$ of the attenuated quantum channel, where the black dashed line, the red solid line, the blue solid line, and the green solid line correspond to the cases that coherent amplitudes are $\alpha = \beta = 0.5, 1.0, 2.0,$ and 3.0, respectively. From Fig.2, capacities are monotonically increasing with respect to the transmissivity. In contrast, in the previous results in which approximate encoding are used, capacities become large when the transmissivity is very small. Owing to using the rigorously realizable encodings, we obtain an reasonable result for all range of the transmissivity. The capacities are smaller than 1 when $\eta < 0.7$ because of double attenuations. In this region, the effect of entanglement vanishes and larger amplitude provides larger capacity. On the other hand, when $\eta > 0.7$, the capacity exceeds the entanglement-unassisted limit (=1[bit]) and smaller amplitude provides larger capacity because of entanglement is more robust in weaker amplitude. Next, we consider the optimum amplitude maximizing the capacity. Figures 3 and 4 show the optimum amplitudes α and β with respect to the transmissivity η, respectively. The optimum values are almost identical for both amplitudes α and β. Note that $|\Psi_4\rangle$ has perfect entanglement when $\alpha = \beta$. When the transmissivity is small, the optimum amplitudes are very large since larger amplitudes provide larger capacity if there is no entanglement assistance. When the transmissivity is large, the optimum amplitudes are very small since smaller amplitudes provide larger capacity if there is entanglement assistance.

Figure 5 shows classical capacities assisted by quasi-Bell states with respect to the transmissivity $\eta_A = \eta_B = \eta$ of the attenuated quantum channel when coherent amplitudes are optimized. From Fig.5, the threshold value of η, that divides entanglement assistance and unassistance, is about 0.7, which is different from the case of *single* loss.

VI. CONCLUSION

We have considered the entanglement-assisted classical communication through an attenuated quantum channel when an attenuated quasi-Bell state is used as shared initial entanglement. We introduce an orthonormal basis and consider rigorously realizable encodings and show properties of entanglement-assisted classical capacity. We find that $I, X, Z,$ and Y correspond to the optimum encoding for an equiprobable classical information when the measurement states of the square-root measurement for the logical qubit states are used as the

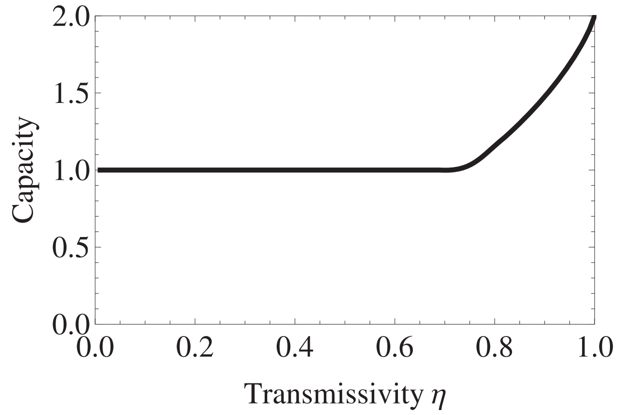

FIG. 5. Classical capacities assisted by quasi-Bell states with respect to the transmissivity $\eta_A = \eta_B = \eta$ of the attenuated quantum channel when coherent amplitudes are optimized.

orthonormal basis. Using the encoding, it turns out that the capacity is attained by a uniform distribution. Unlike the previous result in which an approximate encoding was used, the capacity is monotonically increasing with respect to channel transimissivity. And the threshold value of transimissivity, that divides entanglement assistance and unassistance is about 0.7 for the case of double attenuation, whereas it is 0.5 for single attenuation. In entanglement assistance region, smaller amplitude provides larger capacity. This corresponds that weaker amplitude is more robust against the loss.

In the present paper, we used a maximally entangled coherent state $|\Psi_4\rangle$. Recently, it was shown that there is a case that quantum teleportation using a non-maximally entangled coherent state is superior to that using a maximally entangled coherent state [22]. We will consider the entanglement-assisted classical communication using non-maximally entangled coherent states $|\Psi_1\rangle$ or $|\Psi_3\rangle$.

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