Miniaturization of an Optical Parametric Oscillator with a Bow-Tie Configuration for Broadening a Spectrum of Squeezed Light

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Abstract—A new design for an optical parametric oscillator with a miniaturized bow-tie configuration is proposed and analyzed theoretically by means of a ray matrix. Results show that the cavity length can be reduced from 500 mm which is typical length of a conventional design to 200 mm with maintaining the same optimum beam waist size in an nonlinear optical crystal. The new OPO is supposed to broaden the squeezing spectrum \( \Delta \nu \) from 10 MHz to 26 MHz and sill maintain the high squeezing level.

I. INTRODUCTION

Squeezed states of light is an important resource for quantum information processing for continuous variables [1]. For example, quadrature squeezed vacuum states are applied to realize quantum teleportation which is a fundamental protocol in quantum information processing [2]. The fidelity of such protocols is limited by the squeezing level. So it is very important to generate highly squeezed light to achieve better performance. Recently there is increasing interest in not only squeezing level but a broad spectrum of squeezing, since information processing with high speed is a matter of great importance. Therefore this work focuses on a technical aspect to increase the bandwidth of squeezing spectrum.

A typical method to generate highly squeezed light is utilization of a subthreshold optical parametric oscillator (OPO) which includes a nonlinear optical medium. A performance of the OPO is characterized with several important factors, for example the oscillation threshold of pump power \( P_{th} \) and the escape efficiency \( \rho \). They are described as

\[
P_{th} = \frac{(T + L)^2}{4E_{NL}}
\]

(1)

and

\[
\rho = \frac{T}{T + L}
\]

(2)

respectively, where \( E_{NL} \) is the effective nonlinearity of optical mediums, \( L \) is the intracavity loss, and \( T \) is the transmittance of output coupler. For generating highly squeezed light it is very important to reduce the \( L \) and increase the \( E_{NL} \) in order to improve the \( P_{th} \) and the \( \rho \), and then necessary to optimize the \( T \). On the other hand, the bandwidth of the optical cavity \( \Delta \nu \) can be estimated by using free spectral range \( \nu_{FSR} \) and finesse \( F \), and expressed as

\[
\Delta \nu = \frac{\nu_{FSR}}{F}.
\]

(3)

where

\[
\nu_{FSR} = \frac{c}{\lambda}
\]

(4)

and

\[
F = \pi \sqrt{\frac{(1 - T)(1 - L)}{1 - \sqrt{(1 - T)(1 - L)}}}.
\]

(5)

\( l \) is a total cavity length and \( c \) is the speed of light. It is important to broaden the \( \Delta \nu \) in order to generate broad spectrum of squeezing, since the squeezing spectrum is limited to the \( \Delta \nu \).

Recently the most commonly used OPO is designed as a bow-tie configuration which consists of a nonlinear optical crystal and four mirrors [3], [4], [5], [6] as shown in Fig. 1. Takeno, et al. achieved and \(-9.01 \pm 0.14\) \( \text{dB} \) [5] at 860 nm with a periodically poled KTiOPO\(_4\) (PPKTP) crystal which has rather low losses without blue light induced infrared absorption (BLIIRA). One of the advantages of the bow-tie configuration is its controllability. It is possible to control both a cavity length and phase matching condition independently. Conventional FM side band locking technique is applied to lock the cavity at resonance condition [7]. The phase matching condition is satisfied by controlling a temperature of nonlinear optical crystal. However, there is a problem that squeezing spectrum is limited to rather narrow bandwidth of a bow-tie cavity which is typically around 10MHz [5]. Another excellent method to generate highly squeezed light is a monolithic OPO. Recently Mehmet, et al. succeeded in measuring \(-11.5 \pm 0.1\) dB of squeezing at 1064 nm [8]. They utilized a monolithic OPO of a MgO-7mol% doped LiNbO\(_3\) (MgLN) single crystal whose end surfaces are spherically polished and mirror coated in order to reduce intracavity losses caused by extra optical elements. It was also reported that the MgLN crystal had little losses caused by pump induced absorption at this wavelength. Beside that, they reports broadband spectrum of squeezing over 170 MHz thanks to miniature size of monolithic design with 10 mm of cavity length. However the monolithic OPO has a difficultly to control the cavity
resonance and phase matching condition of nonlinear optical crystal simultaneously. Actually the cavity is not locked in their experiments. Instead, they tune the laser frequency to control the resonance of the OPO. It means that they can not drive more than one OPO at the same time. Experiments of quantum optics usually needs several OPOs simultaneously which are driven by one laser. Therefore the OPO with bow-tie configuration is essentially important from a practical point of view due to its controllability.

In this work a newly designed OPO with a small bow-tie cavity is proposed for the purpose of increasing the bandwidth $\Delta \nu$. The new OPO is supposed to broaden a spectrum of squeezing and still maintain the high squeezing level and the high controllability. The first half of this work reviews the optical design of conventional OPO with bow-tie cavity and the optical calculation with a ray matrix. The latter half deals with the newly designed OPO with rather shorter cavity length and the optical calculation.

II. ANALYSIS OF BOW-TIE CAVITY WITH CONVENTIONAL DESIGN

The optical design of the OPO utilized in previous works is shown in Fig. 1 [4], [5], [6]. The conventional bow-tie cavity has a symmetric configuration with two concave mirrors $M_1$ and $M_2$ with the radius of curvature of $R_1 (=50 \text{ mm})$ and two flat mirrors $M_3$ and $M_4$. The $M_1$, $M_2$ and $M_3$ have high reflectance at 860 nm which is the wavelength of squeezed light. The $M_4$ has partial transmittance typically 10% at 860 nm and used as a coupling mirror. And every mirrors have high transmittance at 430 nm which is the wavelength of pump beam. A nonlinear optical crystal is placed between the two concave mirrors $M_1$ and $M_2$. $l_1$, $l_2$ and $l_3$ are distances between each mirrors and $l_c$ is a length of nonlinear optical crystal as shown by arrows in Fig. 1. Total cavity length $l (= l_1 + 2l_2 + l_3)$ is typically 500 mm. The width of the cavity $d$ should be as minimum as possible in order to suppress cavity folding angles which cause an optical aberration. Typical value of the $d$ is 15 mm which yields about 6° of folding angles.

An optical ray propagation through a variety of optical medium is analyzed by using ray matrices. The design of conventional bow-tie cavity as shown in Fig. 1 is characterized by the $ABCD$ matrix which consists of five square matrices of each optical elements described as Eq. 6.

$$
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix} = 
\begin{pmatrix}
1 & l_2 + \frac{l_3}{2} \\
0 & 1
\end{pmatrix} 
\begin{pmatrix}
1 & 0 \\
-\frac{1}{2R_1} & 1
\end{pmatrix} 
\begin{pmatrix}
1 & \frac{l_3}{2} \\
0 & 1
\end{pmatrix} 
\begin{pmatrix}
1 & 0 \\
0 & \frac{1}{n_1}
\end{pmatrix} 
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} (6)
$$

Each elements of matrix, $A$, $B$, $C$, and $D$ are given by

$$
A = 1 + (l_2 + \frac{l_3}{2})(-\frac{2}{R_1})
$$

$$
B = E + (l_2 + \frac{l_3}{2})F
$$

$$
C = -\frac{2}{R_1}
$$

$$
D = F
$$

where $E$ and $F$ are representing

$$
E = -\frac{l_c}{2} + \frac{l_1 - l_c}{2} \frac{n_1}{n_2}
$$

and

$$
F = -\frac{2}{R_1}E + \frac{n_1}{n_2}
$$

respectively. $n_1$ and $n_2$ are refractive index of the nonlinear optical crystal and the air, and $\lambda$ is wavelength.

The propagation property of a Gaussian beam in an optical cavity can be analyzed with $ABCD$ law. It can be easily shown that the bow-tie cavity has two beam waists. $w_1$ and $w_2$ are the beam waist size in radius at the center of the nonlinear optical crystal and the intermediate between flat mirrors $M_3$ and $M_4$ respectively. The theoretical equation of $w_1$ and $w_2$ are given by using elements of matrix $A$, $B$, $C$, and $D$ as following equations

$$
w_1 = \left( \frac{\lambda}{n_1 \pi} z_1 \right)^{\frac{1}{2}}
$$

and

$$
w_2 = \sqrt{A^2 + \left( \frac{B}{z_1} \right)^2} w_1 = \frac{1}{\sqrt{(Cz_1)^2 + D^2}} \frac{n_1}{n_2} w_1
$$

where $z_1$

$$
z_1 = \left( \frac{AB}{CD} \right)^{\frac{1}{2}}
$$

is corresponding to Rayleigh length of beam waist at the crystal center. Two expressions for the $w_2$ in Eq. (14) are equivalent since there is relation

$$
AD - BC = \frac{n_1}{n_2}
$$

among each matrix elements.

In previous works a periodically poled KTiOPO$_4$ (PP-KTP) crystal with length $l_c$ of 10 mm is used as a
nonlinear optical medium [4], [5]. It is reported that the optimum beam waist size \( w_1 \) in a PPKTP crystal is 0.021 mm which yields the most effective conversion efficiency \( E_{NL} \) of 0.023 (W\(^{-1}\)). At the same time beam waist size \( w_2 \) is about 0.23 mm. So the purpose of this analysis is to calculate optical parameters \( l_1, l_2 \) and \( l_3 \) which realize the optimum waist size. To make a calculation simple, the \( l_2 \) and \( l_3 \) can be expressed as following equations

\[
l_2 = \frac{l}{4} + \frac{d^2}{3l} \tag{17}
\]

and

\[
l_3 = \frac{l}{2} - 2a^2 + l_1. \tag{18}
\]

Notice that \( l_2 \) becomes constant number and \( l_3 \) is a function of only \( l_1 \). These results simplify the calculation process.

Fig. 2 shows the theoretical estimation of the \( w_1 \) and the \( w_2 \) as a function of \( l_1 \), with a set of optical parameters \( l=500 \text{ mm}, l_c=10 \text{ mm}, d=15 \text{ mm}, \lambda=860 \text{ nm}, n_1=1.8396(\text{PPKTP}), n_2=1(\text{air}), \text{and } R_1=50 \text{ mm} \). The optimum beam waist size \( w_1 \) of 0.021 \( \mu \text{m} \) can be achieved when the \( l_1 \) is 57.7 mm. At this time the distances \( l_2 \) and \( l_3 \) are 125.1 mm and 192.2 mm respectively. The estimation of the cavity bandwidth \( \Delta \nu \) from the Eq. 3 with 0.004 of the \( L \) is 10 MHz and shows good agreement with previous experiments [4], [5].

III. PROPOSAL AND ANALYSIS OF A NEWLY DESIGNED BOW-TIE CAVITY

Next purpose of this work is proposal and theoretical analysis of newly designed bow-tie cavity which can generate highly squeezed light with a broad spectrum. From the Eq. 3 one of the possible method to broaden the \( \Delta \nu \) is simply increasing the \( T \). However, a large \( T \) causes a reduction of cavity fineness and an increase of \( P_{th} \). It means a reduction of nonlinear optical interaction and a degradation of squeezing level at same pump power levels. Another solution is to reduce the cavity length \( l \) which yields an increase of the \( \nu_{FSR} \) form the Eq. 4 and broaden the \( \Delta \nu \) form the Eq. 3. However a simple reduction of the \( l \) causes an increasing of the \( w_1 \) which directly acts on the effective nonlinearity \( E_{NL} \), and also degrades a squeezing level. The question is how to miniaturize the bow-tie cavity without changing the optimum beam waist size \( w_1 \) in the nonlinear optical crystal. The answer is using convex mirrors as \( M_3 \) and \( M_4 \) with the radius of curvature of \( R_2 \) which is a minus value instead of two flat mirrors as shown in Fig. 3. Since the convex mirror works as a lens with negative focusing length, the distance between \( M_3 \) and \( M_4 \) can be reduced. This is the fundamental mechanism of this proposal.

The \( ABCD \) matrix is calculated as with previous chapter.

\[
\begin{pmatrix}
A' & B' \\
C' & D'
\end{pmatrix} = \begin{pmatrix}
1 & \frac{1}{2} l_2 \\
0 & 1
\end{pmatrix} \begin{pmatrix}
1 & \frac{1}{2} l_2 \\
0 & 1
\end{pmatrix} \begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix} \begin{pmatrix}
\frac{3}{2} \frac{l_2}{R_2} & \frac{l_3}{R_2} \\
\frac{l_3}{R_2} & \frac{l_2}{R_2}
\end{pmatrix} \begin{pmatrix}
\frac{1}{2} \frac{l_2}{R_2} & \frac{l_3}{R_2} \\
\frac{l_3}{R_2} & \frac{l_2}{R_2}
\end{pmatrix}
\]

\[
\begin{pmatrix}
\frac{1}{2} \frac{l_2}{R_2} & \frac{l_3}{R_2} \\
\frac{l_3}{R_2} & \frac{l_2}{R_2}
\end{pmatrix}
\]

The elements of matrix is expressed as following equations

\[
A' = 1 + (l_2 + \frac{l_3}{2})(- \frac{2}{R_1}) + \frac{l_3}{2}(1 - \frac{2l_2}{R_1})(- \frac{2}{R_2}) \tag{20}
\]

\[
B' = (1 - \frac{l_3}{R_2})E + (l_2 - \frac{l_2l_3}{R_2} + \frac{l_3}{2})F \tag{21}
\]

\[
C' = - \frac{2}{R_1} + (1 - \frac{2l_2}{R_1})(- \frac{2}{R_2}) \tag{22}
\]

\[
D' = - \frac{2}{R_2} + (1 - \frac{2l_2}{R_2})F. \tag{23}
\]

Notice that these equations from Eq. 20 to Eq. 23 become expressions from Eq. 7 to Eq. 10 when the \( R_2 \) goes to \( \infty \) which means the case that \( M_3 \) and \( M_4 \) are flat mirrors.

The new OPO is designed with the fixed cavity length \( l \) of 200 mm and the cavity width \( d \) of 6 mm which can still maintain the cavity folding angles of 6°. Fig. 4 shows calculation results of the \( w_1 \) and the \( w_2 \). About the curvature radius of the mirrors, 38 mm of the \( R_1 \) and -100 mm of the \( R_2 \) which are commercially available values are used for the calculation from a practical point of view.
The result shows that the miniaturized cavity can produce the optimum beam waist size $w_1$ of 0.021 mm when the $l_1$ is 54.9 mm. At this time another beam waist size $w_1$ is 0.105 μm and distances between each mirrors $l_2$ and $l_3$ are 50.2 mm and 44.7 mm respectively. By utilizing newly developed design, the cavity bandwidth $\Delta \nu$ can be broadened up to 26 MHz which is 2.6 times wider than the conventional bow-tie cavity.

**IV. Summary**

The optical cavity with conventional bow-tie configuration is analyzed with the ray matrix. The new design with miniature size is proposed and also analyzed theoretically. The comparison of calculation results and characteristic parameters are summarized in Table. I.

<table>
<thead>
<tr>
<th>Table I</th>
<th>Comparison of characteristic parameters in both type of cavities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cavity length $L$ (mm)</td>
<td>Conventional</td>
</tr>
<tr>
<td>Waist size $w_1$ (mm)</td>
<td>0.021</td>
</tr>
<tr>
<td>Waist size $w_2$ (mm)</td>
<td>0.233</td>
</tr>
<tr>
<td>Cavity bandwidth $\Delta \nu$ (MHz)</td>
<td>10</td>
</tr>
<tr>
<td>$l_1$ (mm)</td>
<td>57.7</td>
</tr>
<tr>
<td>$l_2$ (mm)</td>
<td>125.1</td>
</tr>
<tr>
<td>$l_3$ (mm)</td>
<td>192.2</td>
</tr>
<tr>
<td>$d$ (mm)</td>
<td>15</td>
</tr>
<tr>
<td>$R_1$ (mm)</td>
<td>50</td>
</tr>
<tr>
<td>$R_2$ (mm)</td>
<td>-</td>
</tr>
</tbody>
</table>

Results shows that the cavity length $l$ can be reduced from 500 mm to 200 mm without changing the optimum beam waist size $w_1$ of 0.021 mm. It means that the new OPO can broaden the squeezing spectrum $\Delta \nu$ from 10 MHz to 26 MHz and still maintain the high squeezing level.

**References**