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Error Free Quantum Reading by Quasi Bell State of Entangled Coherent States Part II

–Tolerance to Phase Shift from Optimum Value–

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***Abstract*—A quantum reading of the classical digital memory which was pioneered by Pirandola is one of attractive applications of quantum communication theory. That is, it is a typical example of quantum discrete discrimination problem which provides marvelous results in quantum information science. In the part I, we discovered a possibility of the zero error performance of the quantum reading based on binary phase shift keying by applying entangled coherent state. Recently, Nair et all generalized this result, including M-ary case. In this part II, we investigate the tolerance property of the zero error performance to phase shift from the optimum value such as $\theta = \pi$, and we show that entangled coherent state has a realistic tolerance than other probe states.**

I. INTRODUCTION

The main purpose of the quantum information science is to overcome performances of the classical information technology. Hence, when one considers subjects for quantum information science, the problems that classical information technology cannot cope with has to be chosen. For instance, the quantum communication has to provide a transmission speed beyond 100 Gbit/sec under the average error probability 10^{-9} which is the standard optical communication. In the reading for the optical disk, the quantum reading has to overcome the average error probability 10^{-9} for the tracking speed of several hundred Mbit/sec under very small energy. If proposals of quantum technologies can provide a potential in principle such performances, these may deserve to investigate.

Here, we are concerned with investigation on a potential of quantum reading from digital memories. The classical reading scheme employs a diode laser of coherent state with several hundred photon per signal slot, and the error performance is below 10^{-9} under certain tolerance which corresponds to error free. In addition, they have no trouble in the cost performance. Thus, the current technology in reading scheme of optical disk does not have serious problem. So, the only benefit by employing the quantum technology is that it is applicable to a system in which only a very week illumination of probe light is allowed. However, it is also meaningless if it cannot provide the error free and appropriate tolerance property to ensure it under high speed operation. Thus, the investigations of the tolerance performance are one of the most essential subjects.

In this paper, we analyze the tolerance to phase shift from the optimum value π as serial investigation of part I [1], and we show that the specific entangled coherent state is the optimum in the sense of the tolerance.

II. THEORETICAL MODEL OF QUANTUM READING

A. *Specific model of quantum reading*

There are two types of reading scheme of classical binary memory. These are "Amplitude shift keying (ASK)" and "Phase shift keying(PSK)", respectively. Here we treat only PSK scheme, because PSK scheme can provide the zero error performance[1].

In the conventional reading scheme, the probe state is the single mode coherent state, and the channel is described by

$$\hat{V} = U(\theta) \tag{1}$$

Thus, for 0 or 1 as the information, one can get as follows:

$$|\alpha(0)\rangle = I|\alpha\rangle \tag{2}$$
$$|\alpha(1)\rangle = U(\theta)|\alpha\rangle \tag{3}$$

where I is the identity operator, and θ is a phase shift by information on a disk.

$$U(\theta) = \exp(-i\theta a^\dagger a) \tag{4}$$

where a and $a^\dagger$ are the annihilation and creation operator in bosonic system, respectively.

More general model of quantum reading was proposed by Pirandola [2]. In general, a light source with two modes is employed in the quantum reading, and the channel is described as follows:

$$\hat{V}_{A\otimes B}(\theta) = U_A(\theta) \otimes I_B \tag{5}$$

That is, A-mode has certain unitary operator which corresponds to information on the disk, and B-mode has no interaction. In the above context, many fruitful results have been given by Pirandola and his collaborators[3,4,5,6]. We gave a first example of the possibility of zero error performance in the part I [1]. We here investigate as the next step the tolerance for such a zero error performance, because in general a performance of such a zero error performance is unstable with respect to the target parameter.

B. Ultimate limitation for quantum state discrimination

The formulation of the ultimate detection performance in the quantum system is called quantum detection theory[7]. Nair et al modified the model to the symmetric M-ary quantum reading scheme by phase shift keying [8], following the formulation of [9].

For the binary quantum state discrimination, the general limitation can be evaluated as follows:

$$\begin{aligned} P_e &= \xi_0 Tr \rho_{0(AB)} \Pi_1 + \xi_1 Tr \rho_{1(AB)} \Pi_0 \\ & \quad \Pi_0 + \Pi_1 = I, \quad \Pi_i \geq 0 \end{aligned} \tag{6}$$

where $\{\xi_i\}$ is a priori probability of quantum states $\rho_{i(AB)}$ of the system, $\{\Pi_i\}$ is the detection operator for two mode system, respectively. In the case of pure states $\rho_i = |\Psi_i\rangle\langle\Psi_i|$, the optimum solution is

$$P_e = \frac{1}{2}[1 - \sqrt{1 - 4\xi_0\xi_1|\langle\Psi_0|\Psi_1\rangle|^2}] \tag{7}$$

Thus, the error performance depends strongly on the non-orthogonality between two quantum states.

According to the above formulation, we need to know the inner product property. The inner product between two coherent states is given by

$$|\kappa|^2 = |\langle\alpha|\alpha e^{i\theta}\rangle|^2 = \exp\{-2\alpha^2(1-\cos\theta)\} \tag{8}$$

This gives the theoretical limitation for the conventional reading scheme.

III. Tolerance of error free quantum reading

A. Error free quantum reading with two mode probe state

In the part I, we assumed that $|\Psi\rangle$ of entangled coherent state is employed as the light source or probe state.

$$|\Psi(0)\rangle = h(|\alpha\rangle_A|-\alpha\rangle_B - |-\alpha\rangle_A|\alpha\rangle_B) \tag{9}$$

where $h^2 = 1/(2(1-\exp\{-4\alpha^2\}))$, and where the signal energy is given by

$$N = \frac{2\alpha^2}{\tanh[2\alpha^2]} \tag{10}$$

The A mode is illuminated to a memory disk. The reflection effect $U_A(\theta)$ operates on A mode. When the information signal is 0, θ is 0, and when the signal is 1, the phase shift is θ. The output states are described as follows:

$$\begin{aligned} |\Psi(0)\rangle &= \hat{V}_{A\otimes B}(0) h(|\alpha\rangle_A|-\alpha\rangle_B - |-\alpha\rangle_A|\alpha\rangle_B) \\ &= h(|\alpha\rangle_A|-\alpha\rangle_B - |-\alpha\rangle_A|\alpha\rangle_B),\ \theta = 0, \end{aligned} \tag{11}$$

$$\begin{aligned} |\Psi(\theta)\rangle &= \hat{V}_{A\otimes B}(\theta) h(|\alpha\rangle_A|-\alpha\rangle_B - |-\alpha\rangle_A|\alpha\rangle_B) \\ &= h(|\alpha e^{i\theta}\rangle_A|-\alpha\rangle_B - |-\alpha e^{i\theta}\rangle_A|\alpha\rangle_B) \end{aligned} \tag{12}$$

Thus, the input state $|\Psi(0)\rangle$ is changed to

$$|\Psi(\pi)\rangle = h(|-\alpha\rangle_A|-\alpha\rangle_B - |\alpha\rangle_A|\alpha\rangle_B) \tag{13}$$

when $\theta = \pi$.

The inner product between the above two entangled coherent states corresponding to signals 0 and 1 is

$$\langle\Psi(0)|\Psi(\pi)\rangle = 0 \tag{14}$$

That is, the inner product becomes zero, and it is independent of the energy of light source. This is a crucial property of the entangled coherent state. To investigate the tolerance property, we examine the case of general phase shift as follows:

$$\begin{aligned} |\kappa|^2 &= |\langle\Psi(0)|\Psi(\theta)\rangle|^2 \\ &= \frac{\cosh[2\alpha^2(1+\cos\theta)] - \cos[2\alpha^2\sin\theta]}{2\sinh^2[2\alpha^2]} \end{aligned} \tag{15}$$

Here we define the phase shift from the optimum phase π or $180°$ as follows:

$$\Delta = 2|\theta - 180°| \tag{16}$$

The tolerance can be evaluated by Δ which provides 10^{-9} error free performance. Entangled coherent state, even the signal energy is very small like $N \sim 5.0$, provides

$$\Delta \sim 0.2° \tag{17}$$

for $P_e = 10^{-9}$. When the energy is $N \sim 10$, it provides the completely error free in this Δ.

B. Error free quantum reading with single mode probe state

Recently, Nair et all gave the general theory for quantum reading based on phase shift keying including M-ary quantum signals [8]. Especially their main result shows that the error free quantum reading is possible based on single mode probe system. That is, it was shown that using the probe state

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \tag{18}$$

for reading a binary phase shift encoded memory results in zero error discrimination. The signal energy of this state is given by

$$N = \frac{1}{2} \tag{19}$$

The single mode super position state given by Eq(18) has the following property. That is, the inner product is given by

$$|\kappa|^2 = (1+\cos\theta)/2 \tag{20}$$

This state indeed provides zero error at $\theta = \pi$, but it has no tolerance. For instance, the tolerance property from the optimum value $180°$ is

$$\Delta \sim 0.01° \tag{21}$$

for $P_e = 10^{-9}$. Thus, the error free region is more severe. That is, the error free region is typically pin point which cannot be realized in the real disk.

IV. Necessary conditions for the real applications

As we mentioned in the introduction, the benefit of quantum reading for the classical memory is to provide the weak light illumination system with error free. We have shown the potential for such a requirement. However, it requires to attain the error free at several hundred Mbit/sec (say 100 Mbit/sec). It means that one has to realize the quantum optimum receiver (Doliner Receiver) which operate at 100 Mbit/sec under the error performance below $Pe = 10^{-9}$.

For the communication, the quantum optimum receiver has to operate at 10 G bit/sec under the error performance below $P_e = 10^{-9}$. Recent developments of quantum optimum receiver or sub-optimum receiver [10] are meaningless for claiming the beyond optical communication, because their performance is few bit/sec under error performance of $P_e \sim 0.1$ Thus, following the above example, experimental group should understand such requirements for the real application of quantum reading.

V. Conclusion

The primitive requirement of reading scheme of optical disk is error free at least several hundred bit/sec. In the conventional technology, the scheme is regarded as error free when the average error probability is the below 10^{-9} under certain tolerance. The classical technology can provide such a performance under high speed, but it requires several hundred average photons as the light source. A role of the quantum reading is to provide the same performance under very small energy. In this paper, we have investigated the phase tolerance property of the quantum reading scheme by phase shift keying when the probe state is entangled coherent state. Consequently, we have shown that the appropriate probe state in the sense of tolerance is the specific entangled coherent state. Even it has such a property, it cannot ensure to provide the real application in the real world. We need more careful analysis.

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