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Abstract— **Two-mode squeezed vacuum states are macroscopic quantum entangled states and show nonclassical correlation between quadrature phase amplitudes of each mode. We are interested in applying two-mode squeezed vacuum states to a light source for a quantum radar or quantum imaging system. For that purpose an evaluation method for inseparability of the two-mode squeezed light source is required to verify quantum entanglement. We developed two evaluation methods for inseparability in a lossy optical medium based on analysis using a correlation variance of quadrature phase amplitudes between entangled light beams. One is a method with gain tuning of squeezers in the light source. The other is a method with loss tuning in an optical path of a reference beam.**

I. INTRODUCTION

Quantum illumination is a recently developed target detection method by utilizing quantum entanglement. The theoretical researches have been developed by Massachusetts Institute of Technology [1-3]. In Lloyd's original idea a pair of entangled photons is proposed as a quantum entanglement resource [1]. One of the entangled photons is transmitted towards a target and the other one is safely kept in a radar receiver. The quantum correlation between entangled photons is exploited to improve the error probability of discrimination for target presence or absence even in a lossy and noisy environment. Tan, *et al*. developed quantum illumination with two-mode Gaussian states such as entangled signal and idler beams [2]. Two-mode Gaussian states show better performance in an error probability of the target detection than coherent states. The experimental research was demonstrated by Lopaeva, *et al*. with utilization of entangled signal and idler beams generated by a parametric down conversion process [4]. The experimental results show significant improvement of an error probability even in a lossy and noisy environment compared with classically correlated thermal beams. In these experiments a nonclassicality parameter (called as a generalized Cauchy-Schwarz parameter) is introduced for evaluating photon-number correlation in entangled signal and idler beams.

Quantum imaging, such as ghost imaging, is also an interesting technology for imaging objects with high resolution by utilizing quantum entanglement or classical correlation. Current progress of theoretical and experimental investigations were reviewed and summarized by Shapiro and Boyd [5]. The first experiment of ghost imaging was demonstrated by Pittman, *et al*. [6] by utilizing a pair of entangled photons generated by spontaneous parametric down conversion and coincidence detection of two photons. Meyers, *et al*. demonstrated ghost imaging of objects by measuring reflected photons with a thermal light source [7] and also reported turbulence-free ghost imaging [8].

Two-mode squeezed vacuum states are macroscopic quantum entangled states of an electro-magnetic field and have nonclassical correlation between quadrature phase amplitudes. We are interested in applying two-mode squeezed vacuum states to a light source for a quantum radar system which is based on quantum illumination method or quantum imaging system. For that purpose it is necessary to develop not only a light source with experimental techniques of quantum optics but also an evaluation method for quantum entanglement with a theoretical approach. Recently two-mode squeezed light source was developed for generating entangled images by means of a four wave mixing process in rubidium vapor which is a third-order nonlinear interaction [9]. We also started to construct a continuous-wave two-mode squeezed light source by utilizing sub-threshold optical parametric oscillators with rather strong second-order nonlinearity [10]. In the actual quantum radar or quantum imaging system an optical beam is transmitted through a turbulent atmosphere or a lossy optical medium which degrades quantum entanglement of a light source and affects performance of target detection. So, it is required to investigate the evaluation method in such environments for developing a quantum radar system. An optical loss is one of the simplest and most probable examples of environmental factors. In this work we developed the evaluation method for quantum entanglement in two-mode squeezed vacuum states with consideration for an optical loss.

One important measure to evaluate quantum entanglement is logarithmic negativity [11-13]. In previous experiments for demonstrating quantum distillation, logarithmic negativity was used for evaluating two-mode squeezed vacuum states [14,15]. Another useful and practical method is an inseparability criterion for continuous-variable entangled states developed by Duan, *et al*. [16] and Simon [17]. This criterion is a sufficient condition of quantum entanglement and frequently used in continuous-variable quantum-optics experiments to verify nonclassical correlation [18]. This is based on a correlation variance of quadrature phase amplitudes which can be directly obtained by balanced homodyne measurement. In this work we studied the correlation variance of two-mode squeezed vacuum states in a lossy optical medium and develop the evaluation method to verify quantum entanglement.

II. TWO-MODE SQUEEZED VACUUM STATES AND INSEPARABILITY CRITERION

In this section we briefly review the description of two-mode squeezed states with Heisenberg representation as is often used in continuous-variable quantum-optics experiments [18]. Entangled two-mode squeezed vacuum states are often generated by combining two single-mode squeezed light beams with a beam splitter as shown in Figure 1. Two single-mode squeezed light beams SQ_1 and SQ_2 are generated by two-independent squeezers $S(r)$ and $S(-r)$ which act on vacuum states Vac_1 and Vac_2, respectively. Here squeezing parameter r corresponds to the optical gain of squeezers and is an important experimental parameter to control a squeezing level of incident squeezed light beams SQ_1 and SQ_2. A relative optical phase between incident squeezed light beams SQ_1 and SQ_2 is kept at 90 degrees. So, we write complex amplitude operators of input squeezed light beams SQ_1 and SQ_2 as

$$\hat{a}_1 = e^{-r}\hat{x}_1 + ie^{r}\hat{y}_1, \tag{1}$$

$$\hat{a}_2 = e^{r}\hat{x}_2 + ie^{-r}\hat{y}_2 \tag{2}$$

where $\hat{x}_i$ and $\hat{y}_i$ are quadrature phase amplitude operators of input vacuum state Vac_i (i=1,2) for each squeezer. We assume that two squeezers are characterized by same squeezing parameter r and both squeezed light beams SQ_1 and SQ_2 are in pure states. After combining two single-mode squeezed light beams SQ_1 and SQ_2 by beam splitter BS(0.5) with transmissivity of 0.5, complex amplitude operators of entangled output beams Out_1 and Out_2 are given by

$$\hat{A}_1 = \hat{X}_1 + i\hat{Y}_1 = \frac{1}{\sqrt{2}}(e^{-r}\hat{x}_1 + e^{r}\hat{x}_2) + \frac{i}{\sqrt{2}}(e^{-r}\hat{y}_1 + e^{r}\hat{y}_2), \tag{3}$$

$$\hat{A}_2 = \hat{X}_2 + i\hat{Y}_2 = \frac{1}{\sqrt{2}}(-^{-r}\hat{x}_1 + e^{r}\hat{x}_2) + \frac{i}{\sqrt{2}}(-e^{-r}\hat{y}_1 + e^{r}\hat{y}_2), \tag{4}$$

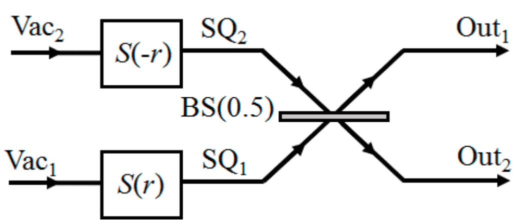

Figure 1 Entangled two-mode squeezed light beams Out_1 and Out_2 are generated by combining two single-mode squeezed light beams SQ_1 and SQ_2 with beam splitter BS(0.5). The squeezed light beams SQ_1 and SQ_2 are generated by squeezers S(r) and S($-r$) which act on vacuum states Vac_1 and Vac_2. Both squeezers are characterized by squeezing parameter r which corresponds to the optical gain of squeezers.

respectively.

To verify quantum entanglement in two-mode squeezed light source an inseparability criterion is required. For that purpose a correlation variance $\Delta^2_{1,2}$ in quadrature phase amplitudes between two output beams Out_1 and Out_2 is calculated by following equation

$$\Delta^2_{1,2} = \langle [\Delta(\hat{X}_1 - \hat{X}_2)]^2 + [\Delta(\hat{Y}_1 + \hat{Y}_2)]^2 \rangle. \tag{5}$$

It has been proven by Duan, *et al.* [11] and Simon[12] that two outputs Out_1 and Out_2 are inseparable and show quantum entanglement when

$$\Delta^2_{1,2} < 1. \tag{6}$$

So, Eq.(6) is called the inseparability criterion. In the case of two-mode squeezed vacuum states, correlation variance $\Delta^2_{1,2}$ can be calculated by using Eq.(3) and (4) and given by a function of squeezing parameter r as following equation

$$\Delta^2_{1,2} = e^{-2r}. \tag{7}$$

It can be easily seen that $\Delta^2_{1,2}$ is less than one and satisfies the inseparability criterion given by Eq.(6) as long as squeezing parameter r is greater than zero. The value of Eq.(7) goes to zero in the case with an infinite value of squeezing parameter r. On the other hand the value of Eq.(7) is one when squeezing parameter r is zero. It means that squeezers don't act on vacuum states Vac_1 and Vac_2, so there is no quantum correlation between final outputs Out_1 and Out_2.

III. CORRELATION VARIANCE IN A LOSSY OPTICAL MEDIUM

In this section we precisely studied the correlation variance in a lossy optical medium to develop the evaluation method for two-mode squeezed vacuum states. In previous experimental works the inseparability was studied with consideration for an optical loss equally affecting on two entangled light beams [15, 19]. In quantum radar experiments, however, one of the entangled beams is transmitted towards a target in a lossy medium and the other beam is safely kept in a radar receiver. So, it is essentially required to derive the correlation variance with consideration for an asymmetric optical loss. For that purpose, two different optical losses L_1 and L_2 were introduced on the optical paths for output beams Out_1 and Out_2 as shown in Figure 2. Then a more general expression for the correlation variance is derived as a function of optical losses L_1 and L_2. In quantum optics an optical loss is modeled as mixing of a vacuum state through a beam splitter. The complex amplitude operator of output $\hat{A}_i$ is affected by optical loss L_i and then becomes $\hat{A}_i{}' = \sqrt{1-L_i}\hat{A}_i + \sqrt{L_i}\hat{a}_i{}'$ (i=1, 2). $\hat{a}_i{}' = \hat{x}_i{}' + i\hat{y}_i{}'$ is the complex amplitude operator of vacuum state $Vac_i{}'$ which is mixed with the output modes through the beam splitter $BS(1-L_i)$ with transmissivity of $1-L_i$ (i=1, 2). Finally the complex amplitude operators in entangled outputs $Out_1{}'$ and $Out_2{}'$are given as

$$\hat{A}_1{}' = \hat{X}_1{}' + i\hat{Y}_1{}' = \frac{\sqrt{1-L_1}}{\sqrt{2}}(e^{-r}\hat{x}_1 + e^{r}\hat{x}_2) + \sqrt{L_1}\hat{x}_1' + i\left\{\frac{\sqrt{1-L_1}}{\sqrt{2}}(e^{-r}\hat{y}_1 + e^{r}\hat{y}_2) + \sqrt{L_1}\hat{y}_1{}'\right\}, \tag{8}$$

$$\hat{A}_2{}' = \hat{X}_2{}' + i\hat{Y}_2{}' = \frac{\sqrt{1-L_2}}{\sqrt{2}}(-e^{-r}\hat{x}_1 + e^{r}\hat{x}_2) + \sqrt{L_2}\hat{x}_2'$$

$$+i\left\{\frac{\sqrt{1-L_2}}{\sqrt{2}}\left(-e^{-r}\hat{y}_1+e^{r}\hat{y}_2\right)+\sqrt{L_2}\hat{y}_2{}'\right\}, \qquad (9)$$

respectively. Then the correlation variance of Eq.(7) is modified as

$$\Delta^2_{1,2}(r,L_1,L_2) = \frac{e^{-2r}}{4}\left(\sqrt{1-L_1}+\sqrt{1-L_2}\right)^2 + \frac{e^{2r}}{4}\left(\sqrt{1-L_1}-\sqrt{1-L_2}\right)^2 + \frac{1}{2}(L_1+L_2) \qquad (10)$$

and given as a function of squeezing parameter r and the optical loss L_1 and L_2.

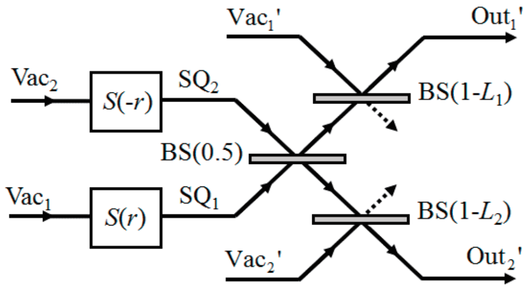

Figure 2 Effects of optical losses L_1 and L_2 for two-mode squeezed vacuum states. The optical losses in the path of output modes are modeled as mixing of vacuum states $\mathrm{Vac}_1{}'$ and $\mathrm{Vac}_2{}'$ through beam splitters $\mathrm{BS}(1-L_1)$ and $\mathrm{BS}(1-L_2)$.

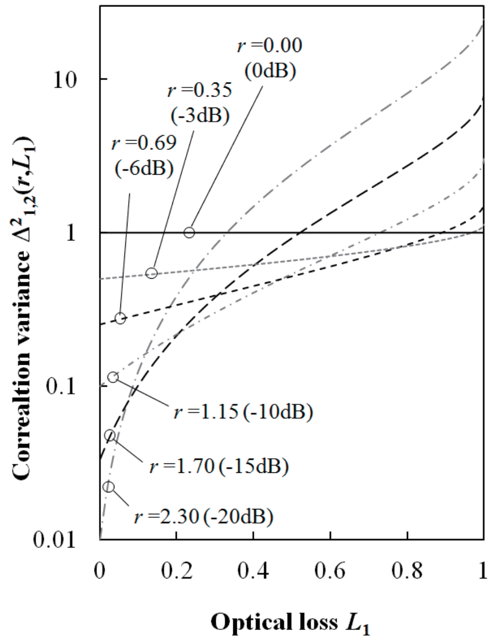

Figure 3 Calculation results of the correlation variance $\Delta^2_{1,2}(r,L_1)$ in case with the asymmetric configuration of the optical loss. Only optical loss L_1 is considered and L_2 is assumed to be zero. Squeezing parameter r is set at 2.30, 1.70, 1.15, 0.69, 0.35, and 0.

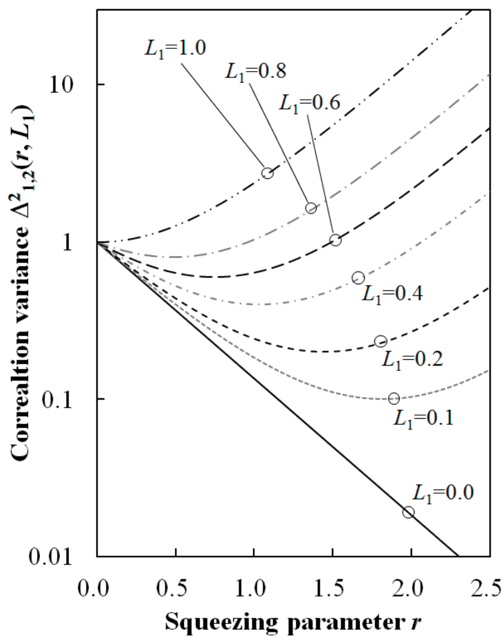

Figure 4 Calculation results of the correlation variance $\Delta^2_{1,2}(r,L_1)$ in case with the asymmetric configuration of the optical loss. Squeezing parameter r is continuously tuned at certain values of optical loss L_1.

IV. Case 1. An evaluation method with gain tuning

In an actual quantum radar system one beam Out_1 is used for a signal beam and emitted to a target which exists in a medium with optical loss L_1. The other beam Out_2 is used as a reference beam and safely restored in a radar receiver. For the latter beam Out_2, optical loss L_2 can be assumed to be zero. In this asymmetric configuration of the optical loss the correlation variance $\Delta^2_{1,2}$ becomes

$$\Delta^2_{1,2}(r,L_1) = \frac{e^{-2r}}{4}\left(\sqrt{1-L_1}+1\right)^2 + \frac{e^{2r}}{4}\left(\sqrt{1-L_1}-1\right)^2 + \frac{L_1}{2}. \qquad (11)$$

Figure 3 shows calculation results of the correlation variance $\Delta^2_{1,2}(r,L_1)$ using Eq.(11) as a function of optical loss L_1 with certain values of squeezing parameter r (=2.30, 1.70, 1.15, 0.69, 0.35, and 0.00). Squeezing parameter r is related with a squeezing level of incident squeezed light beams SQ_1 and SQ_2 and calculated by using a relation of $10\log e^{-2r}$(dB). The asymmetric optical loss is serious problem, since it degrades quantum entanglement especially in the case with large squeezing parameter r. It is obvious that the correlation variance drastically increases and does not satisfy the inseparability criterion of $\Delta^2_{1,2} < 1$ at high values of optical loss L_1. On the other hand in the case with rather small squeezing parameter r, the correlation variance shows relatively high resistance and satisfy the inseparability criterion of $\Delta^2_{1,2} < 1$ in broad range of the values of optical loss L_1. In Figure 3 the correlation variance seems to take its minimum value by changing the squeezing parameter r at certain optical loss L_1. To confirm this phenomenon we performed another calculation using Eq.(11). Figure 4 shows calculation results of the correlation variance by tuning squeezing parameter r (optical gain of squeezers) at certain values of optical loss L_1. It is obvious that the inseparability criterion $\Delta^2_{1,2} < 1$ is maintained by gain tuning method in response to changes in the optical loss L_1. In this evaluation method the asymmetric configuration of the optical loss is compensated by reducing squeezing parameter r. In order to develop a quantum radar or quantum imaging system, it is essentially important to verify quantum entanglement of the light source in the asymmetric configuration of the optical loss. So, the gain tuning is a practical and useful scheme for the evaluation method in the asymmetric configuration of the optical loss. In actual experiments squeezing parameter r can be easily controlled, for example, by changing a pump power level for squeezers.

V. CASE 2. AN EVALUATION METHOD WITH LOSS TUNING

Next we considered another evaluation method. The asymmetric optical loss is also compensated by varying optical loss L_2. For that purpose the controllable optical loss L_2 is added on the optical path of reference beam Out_2 by artificial means and adjusted to coincide with optical loss L_1 in outer environments. In actual experiments the controllable optical loss can be easily prepared by using a half wave plate and a polarization beam splitter on the optical path of the reference beam Out_2. In this optical configuration the optical loss L_2 is assumed as $\varepsilon * L_1$ where ε is the loss tuning parameter $(0 \le \varepsilon \le 1)$. The correlation variance given by Eq.(10) becomes

$$\Delta^2_{1,2}(r,\varepsilon,L_1) = \frac{e^{-2r}}{4}\left(\sqrt{1-L_1}+\sqrt{1-\varepsilon L_1}\right)^2 + \frac{e^{2r}}{4}\left(\sqrt{1-L_1}-\sqrt{1-\varepsilon L_1}\right)^2 + \frac{1}{2}(L_1+\varepsilon L_1) \quad (12)$$

Figure 5 shows calculation results of the correlation variance using Eq.(12) varying loss tuning parameter ε (= 0.0, 0.5, 0.8, 0.9, and 1.0) at fixed squeezing parameter r of 2.30. It is obvious that the correlation variance decreases by varying loss tuning parameter ε from 0.0 to 1.0. And it is possible to maintain the inseparability criterion $\Delta^2_{1,2} < 1$ even in high values of optical loss L_1. So, the loss tuning is also useful scheme for the evaluation method to verify quantum entanglement in two-mode squeezed vacuum states in the asymmetric configuration of the optical loss.

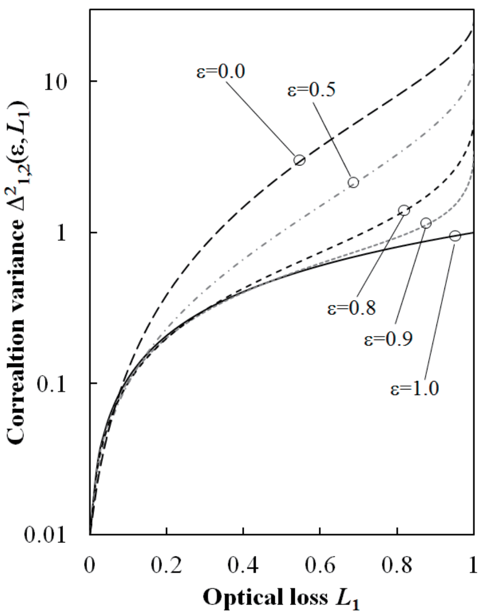

Figure 5 Calculation results of the correlation variance $\Delta^2_{1,2}(r,L_1)$ in case with the asymmetric configuration of the optical loss. Optical loss L_2 is varied by loss tuning parameter ε. Squeezing parameter r is fixed to 2.30.

Next we considered an ideal case where optical loss L_2 is perfectly matching with optical loss L_1 (ε =1.0). In this symmetric optical configuration, the same amount of optical loss L (= L_1 = L_2) affects on both of the entangled beams. The correlation variance is simplified and given as a function of squeezing parameter r and symmetric optical loss L

$$\Delta^2_{1,2}(r,L) = e^{-2r}(1-L)+L. \quad (13)$$

The expression of Eq.(13) is consistent with the previously reported formula for the correlation variance [15, 19]. Figure 6 shows calculation results of the correlation variance with the symmetric optical loss L with certain values of squeezing parameter r (=2.30, 1.70, 1.15, 0.69, 0.35, and 0.00). The values of the correlation variance increase due to optical loss L. However they are always less than one at any optical loss except L=1 as long as squeezing parameter r is greater than zero. It means that the correlation variance always satisfies the inseparability criterion $\Delta^2_{1,2} < 1$ even in the case with small squeezing parameter r, if optical loss L_2 is perfectly matching with optical loss L_1.

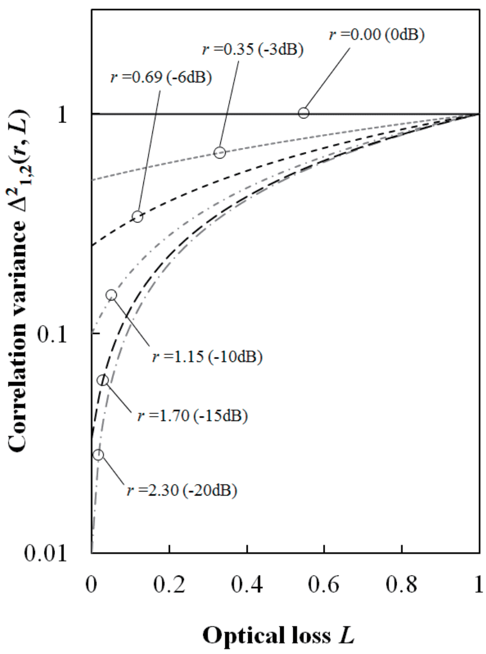

Figure 6 Calculation results of the correlation variance $\Delta^2_{1,2}(r,L)$ in case with the symmetric optical loss L_1=L_2=L.

VI. CONCLUSION

We developed evaluation methods for inseparability criterion to verify quantum entanglement of two-mode squeezed vacuum states in the asymmetric configuration of the optical loss. The asymmetric loss is serious problem in an actual quantum radar or quantum imaging system, since it drastically degrades quantum entanglement. We proposed two evaluation methods in this work. One is a gain tuning technique and the other is a loss tuning technique. Both methods are effectively used to verify quantum entanglement in the asymmetric configuration of the optical loss.

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