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***Abstract*—Two-mode squeezed vacuum states are important for realizing quantum illumination radar because of their non-classical correlation. In quantum illumination radar one of the entangled beams is transmitted to a target in a lossy optical medium as a signal beam. The other beam called as a reference is send to the radar receiver with a lossless channel. So, the entangled light beams are exposed to such an asymmetric optical-loss condition. This causes violation of their inseparability criterion for non-classical correlation.**

In this article we assume an asymmetric two-mode squeezed vacuum states where the signal beam has larger quadrature phase amplitudes and average photon number than the reference beam. We calculate a correlation variance of quadrature phase amplitudes between entangled beams in the asymmetric optical-loss condition. And then we check the non-classical correlation of the light source based on Duan's and Simon's inseparability criterion. We could find out some conditions where asymmetric property of two-mode squeezed vacuum states has an ability to compensate the effect of the asymmetric optical-loss and can maintain the inseparability criterion.

I. Introduction

Quantum illumination is a recently developed target detection technique utilizing quantum entanglement [1], [2], [3]. In this technique quantum entanglement is exploited to improve error probability of discrimination for target presence or absence even in a lossy and noisy environment. Initially a pair of entangled photons was proposed as a quantum entanglement resource [1]. After that, Tan, *et al*. proposed entangled two-mode Gaussian states such as two-mode squeezed vacuum states as a light source and showed improvement in error probability of target detection [2]. So, the author is interested in applying two-mode squeezed vacuum states as a light source to an experimental study of quantum illumination [4], [5].

Two-mode squeezed vacuum states are macroscopic quantum entangled states of electro-magnetic fields and have non-classical correlation between quadrature phase amplitudes in each optical beam. It is usually generated by mixing two independent single-mode squeezed vacuum states characterized by same squeezing parameter r. They are combined by a beam splitter with transmissivity of 0.5 with a relative optical phase of $90°$. In this case the entangled output beams have same quadrature phase amplitude noises and average photon number. In this article, therefore, we call them symmetric two-mode squeezed vacuum states. To verify non-classical correlation of a light source, an inseparability criterion is necessary. One useful and practical inseparability criterion for continuous-variable entangled states was developed by Duan, *et al*. [6] and Simon [7]. This criterion is based on a correlation variance between quadrature phase amplitudes of entangled light beams. The correlation variance can be directly detected by balanced homodyne measurement. Therefore it has been frequently used in continuous-variable quantum-optics experiments to verify quantum entanglement [8].

In the quantum illumination radar, two-mode squeezed vacuum states are exposed to an asymmetric optical loss. One of the entangled beams usually called as a signal beam is transmitted towards a target through a turbulent atmosphere or a lossy optical medium. The other beam usually called as a reference beam is directly sent to the radar receiver with a lossless channel. In a previous work the author studied how non-classical correlation of symmetric two-mode squeezed vacuum states are affected in such an asymmetric optical loss [9]. In these optical conditions only the signal beam is affected by the optical loss and attenuates its quadrature phase amplitude, and finally decreases the average photon number. As a result symmetric two-mode squeezed vacuum states become asymmetric states. The asymmetric property of the light source causes increasing of the correlation variance of quadrature phase amplitudes and then violates the inseparability criterion.

In this article we consider asymmetric two-mode squeezed vacuum states as a light source for the quantum illumination radar. In these asymmetric states the signal beam has a larger quadrature phase amplitude and average photon number than the reference beam. Then the correlation variance of them are more than one and violates the inseparability criterion. However, when only the signal beam is exposed to an optical loss, its quadrature phase amplitudes and average photon number are reduced and finally expected to become comparable to those of the reference beam . As a result it is expected that asymmetric property of the light source is compensated and then its non-classical correlation is maintained. We calculate the correlation variance of quadrature phase amplitudes of asymmetric squeezed vacuum states after exposition to asymmetric optical-loss conditions. And we check the inseparability criterion to study non-classical correlation of the light source.

II. Optical model

In this chapter we derive a description of asymmetric two-mode squeezed vacuum states using Heisenberg representation (with unit of $\hbar = 1/2$, i.e. $\left[\hat{x}, \hat{y}\right] = i/2$) as is often used in continuous-variable quantum-optics [8]. Fig. 1 shows an optical model to generate asymmetric two-mode squeezed vacuum states. In this model two single-mode squeezed vacuum states SQ_1 and SQ_2 are combined with a beam splitter BS. A relative optical phase between squeezed vacuum states SQ_1 and SQ_2 is fixed at $90°$. In this article we derive the description of two-mode squeezed vacuum states for not only the symmetric case but also an asymmetric case. So, we assume that beam splitter BS has variable transmittance T and incident two single-mode squeezed vacuum states SQ_1 and SQ_2 are characterized by squeezing parameter r_1 and r_2, respectively. Squeezing parameter r_i corresponds to optical gain of a squeezer $SQ_i(r_i)$ and related with average photon number n_i of the incident squeezed vacuum state SQ_i by an equation of $n_i = \sinh^2 r_i$ $(i = 1, 2)$.

Firstly we write complex amplitude operators of input squeezed vacuum states SQ_1 and SQ_2 as

$$\hat{a}_1 = e^{-r_1}\hat{x}_1 + ie^{r_1}\hat{y}_1 \tag{1}$$

and

$$\hat{a}_2 = e^{r_2}\hat{x}_2 + ie^{-r_2}\hat{y}_2. \tag{2}$$

$\hat{x}_i$ and $\hat{y}_i$ $(i = 1, 2)$ are quadrature phase amplitude operators of incident vacuum states for two independent squeezers which are not shown in Fig. 1. After mixing two single-mode squeezed vacuum states SQ_1 and SQ_2 by the beam splitter with transmittance T, entangled two-mode output states Out_1 and Out_2 are generated. They corresponds to signal and reference beams, respectively. The complex amplitude operators of two outputs Out_1 and Out_2 are calculated as

$$\begin{aligned}\hat{A}_1 &= \hat{X}_1 + i\hat{Y}_1\\ &= \left(\sqrt{T}e^{-r_1}\hat{x}_1 + \sqrt{1-T}e^{r_2}\hat{x}_2\right)\\ &\quad + i\left(\sqrt{T}e^{r_1}\hat{y}_1 + \sqrt{1-T}e^{-r_2}\hat{y}_2\right)\end{aligned} \tag{3}$$

and

$$\begin{aligned}\hat{A}_2 &= \hat{X}_2 + i\hat{Y}_2\\ &= \left(-\sqrt{1-T}e^{-r_1}\hat{x}_1 + \sqrt{T}e^{r_2}\hat{x}_2\right)\\ &\quad + i\left(-\sqrt{1-T}e^{r_1}\hat{y}_1 + \sqrt{T}e^{-r_2}\hat{y}_2\right).\end{aligned} \tag{4}$$

The average photon number n_1 and n_2 of outputs Out_1 and Out_2 are calculated as

$$n_1 = T\sinh^2 r_1 + (1-T)\sinh^2 r_2 \tag{5}$$

and

$$n_2 = (1-T)\sinh^2 r_1 + T\sinh^2 r_2, \tag{6}$$

respectively. When squeezing parameters r_1 and r_2 are equal (r_1=r_2=r), average photon numbers n_1 and n_2 of output states become same as $\sinh^2 r$. On the other hand, as long as beam-splitter transmittance is kept at 0.5, average photon numbers n_1 and n_2 of outputs are also same and given as $0.5\sinh^2 r_1 + 0.5\sinh^2 r_2$. So, in both cases, two outputs Out_1 and Out_2 are in symmetric two-mode states. To generate asymmetric two-mode squeezed vacuum states where the average photon number n_1 is larger than n_2, it is required to chose optical conditions with $r_1 > r_2$ and $0.5 < T < 1$, or $r_1 < r_2$ and $0 < T < 0.5$. In this article we chose the former condition.

In order to study the effect of asymmetric optical-loss conditions on two-mode squeezed vacuum states, we introduce optical losses L_1 and L_2 in both optical paths of output beams Out_1 and Out_2, respectively, as shown in Fig. 1. These optical losses are modeled as mixture of vacuum states through beam splitters with amplitude transmittance of $\sqrt{1-L_1}$ and $\sqrt{1-L_2}$, respectively. To describe an asymmetric optical-loss condition for quantum illumination radar, it is reasonable to assume that optical loss L_2 is zero and only L_1 is varied as a calculation parameter. To describe the effect of optical losses we use quadrature phase amplitude operators of vacuum states Vac_1 ($\hat{x}_1'$, $\hat{y}_1'$) and Vac_2 ($\hat{x}_2'$, $\hat{y}_2'$) which come to be mixed in the output states Out_1 and Out_2, respectively. Finally complex amplitude operators of output beams Out_1' and Out_2' after exposition to the asymmetric optical-loss condition are given as

$$\begin{aligned}\hat{A}_1' &= \hat{X}_1' + i\hat{Y}_1'\\ &= \sqrt{1-L_1}\left(\sqrt{T}e^{-r_1}\hat{x}_1 + \sqrt{1-T}e^{r_2}\hat{x}_2\right) + \sqrt{L_1}\hat{x}_1'\\ &\quad + i\left\{\sqrt{1-L_1}\left(\sqrt{T}e^{r_1}\hat{y}_1 + \sqrt{1-T}e^{-r_2}\hat{y}_2\right) + \sqrt{L_1}\hat{y}_1'\right\}\end{aligned} \tag{7}$$

and

$$\begin{aligned}\hat{A}_2' &= \hat{X}_2' + i\hat{Y}_2'\\ &= \sqrt{1-L_2}\left(-\sqrt{1-T}e^{-r_1}\hat{x}_1 + \sqrt{T}e^{r_2}\hat{x}_2\right) + \sqrt{L_2}\hat{x}_2'\\ &\quad + i\left\{\sqrt{1-L_2}\left(-\sqrt{1-T}e^{r_1}\hat{y}_1 + \sqrt{T}e^{-r_2}\hat{y}_2\right) + \sqrt{L_2}\hat{y}_2'\right\}.\end{aligned} \tag{8}$$

The average photon number of outputs is also affected by the asymmetric optical loss. The average photon number n_1' and n_2' of outputs Out_1' and Out_2' are calculated as

$$n_1' = (1-L_1)\left\{T\sinh^2 r_1 + (1-T)\sinh^2 r_2\right\} \tag{9}$$

and

$$n_2' = (1-L_2)\left\{(1-T)\sinh^2 r_1 + T\sinh^2 r_2\right\}, \tag{10}$$

respectively.

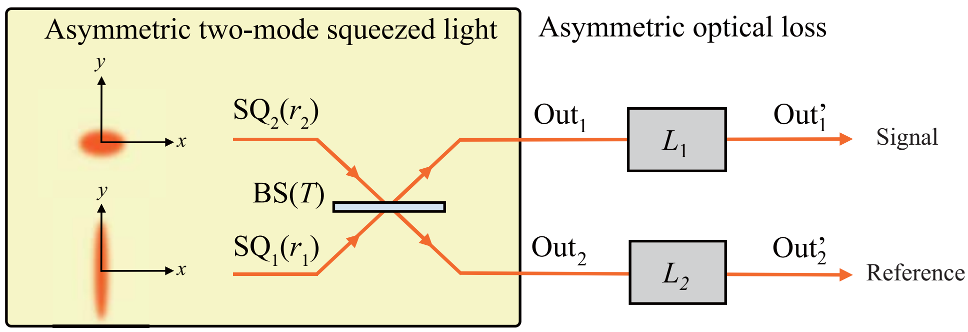

Fig. 1. Optical model for generating asymmetric two-mode squeezed vacuum states. Incident single-mode squeezed vacuum states SQ_1 and SQ_2 characterized by squeezing parameter r_1 and r_2, respectively, are combined using beam splitter BS(T) with transmittance of T. A relative optical phase between them is fixed at 90°. Two entangled outputs Out_1 and Out_2 are affected by an environment with optical losses L_1 and L_2 in each optical path and become final outputs Out_1' and Out_2', respectively.

Correlation variance of quadrature phase amplitudes between outputs Out_1' and Out_2' is defined as

$$\Delta_{1,2}{}^2 = \langle\left[\Delta\left(\hat{X}_1' - \hat{X}_2'\right)\right]^2\rangle + \langle\left[\Delta\left(\hat{Y}_1' + \hat{Y}_2'\right)\right]^2\rangle. \quad (11)$$

It has been proven by Duan, *et al.* [6] and Simon [7] that two outputs Out_1' and Out_2' are inseparable and show quantum entanglement when the relation

$$\Delta_{1,2}{}^2 < 1 \quad (12)$$

is satisfied. Equation (12) is called as an inseparability criterion and corresponds to a sufficient condition of non-classical correlation. Therefore the quantitative relation between correlation variance $\Delta_{1,2}{}^2$ and the strength of non-classical correlation is not clear with this. Recently a measure for quantifying quantum entanglement of two-mode squeezed vacuum states is reported by Tserkis, *et al.* [10] However it is still difficult to deal with two-mode squeezed vacuum states affected by optical losses. So, in this work we use the inseparability criterion to study and check non-classical correlation of the light source.

In our optical model, correlation variance $\Delta_{1,2}{}^2$ can be calculated using quadrature phase amplitudes of final output states Out_1' and Out_2'. The first and second term of correlation variance $\Delta_{1,2}{}^2$ are calculated as

$$\begin{aligned}&\langle\left[\Delta\left(\hat{X}_1 - \hat{X}_2\right)\right]^2\rangle\\ &\quad= \frac{1}{4}(\sqrt{1-L_1}\sqrt{T} + \sqrt{1-L_2}\sqrt{1-T})^2 e^{-2r_1}\\ &\quad+ \frac{1}{4}(\sqrt{1-L_1}\sqrt{1-T} - \sqrt{1-L_2}\sqrt{T})^2 e^{2r_2}\\ &\quad+ \frac{1}{4}(L_1 + L_2),\end{aligned} \quad (13)$$

$$\begin{aligned}&\langle\left[\Delta\left(\hat{Y}_1 + \hat{Y}_2\right)\right]^2\rangle\\ &\quad= \frac{1}{4}(\sqrt{1-L_1}\sqrt{T} - \sqrt{1-L_2}\sqrt{1-T})^2 e^{2r_1}\\ &\quad+ \frac{1}{4}(\sqrt{1-L_1}\sqrt{1-T} + \sqrt{1-L_2}\sqrt{T})^2 e^{-2r_2}\\ &\quad+ \frac{1}{4}(L_1 + L_2)\end{aligned} \quad (14)$$

and are given as a function of squeezing parameters r_1, r_2, beam-splitter transmittance T, and optical losses L_1 and L_2.

III. Calculation Results

Firstly we prepare asymmetric two-mode squeezed vacuum states Out_1 and Out_2 as an incident light source. For that purpose it is require to set squeezing parameter r_2 smaller than squeezing parameter r_1 and simultaneously vary beam-splitter transmittance T more than 0.5. Notice that two-mode output Out_1 and Out_2 have a symmetric structure as long as beam-splitter transmittance T is 0.5 even with the set of different squeezing parameters r_1 and r_2. To study the non-classical property of asymmetric two-mode squeezed vacuum states after exposition to an asymmetric optical loss, we calculated correlation variance $\Delta_{1,2}{}^2$ of the final output states Out_1' and Out_2'.

Fig. 2 shows calculation results of correlation variance $\Delta_{1,2}{}^2$ as a function of squeezing parameter r_2 with beam-splitter transmittance T at 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0, respectively. Optical loss L_2 was set at zero and L_1 was set at (a) 0.0, (b) 0.5, and (c) 0.9. Squeezing parameter r_1 of incident squeezed vacuum state SQ_1 was fixed at 1.5. In Fig. 2 (a) with optical loss L_1 of 0.0, correlation variances $\Delta_{1,2}{}^2$ monotonically increase when beam splitter transmittance T increases. So, the asymmetric property of two-mode squeezed vacuum states is only making its correlation variance $\Delta_{1,2}{}^2$ large under the lossless condition (L_1=L_2=0).

In Fig. 2 (b) with optical loss L_1 of 0.5, correlation variances $\Delta_{1,2}{}^2$ with T=0.5 and 0.6 are larger than those of Fig. 2 (a) at the full range of squeezing parameter r_2. Correlation variance $\Delta_{1,2}{}^2$ with T=0.6 shows lower values than that with T=0.5 when squeezing parameter r_2 is less than 1.206 and gives the minimum value of 0.503 at r_2 of 0.658. On the other hand correlation variances $\Delta_{1,2}{}^2$ with T=0.7, 0.8, 0.9, and 1.0 decrease as compared with those of Fig. 2 (a) especially at low values of squeezing parameter r_2.

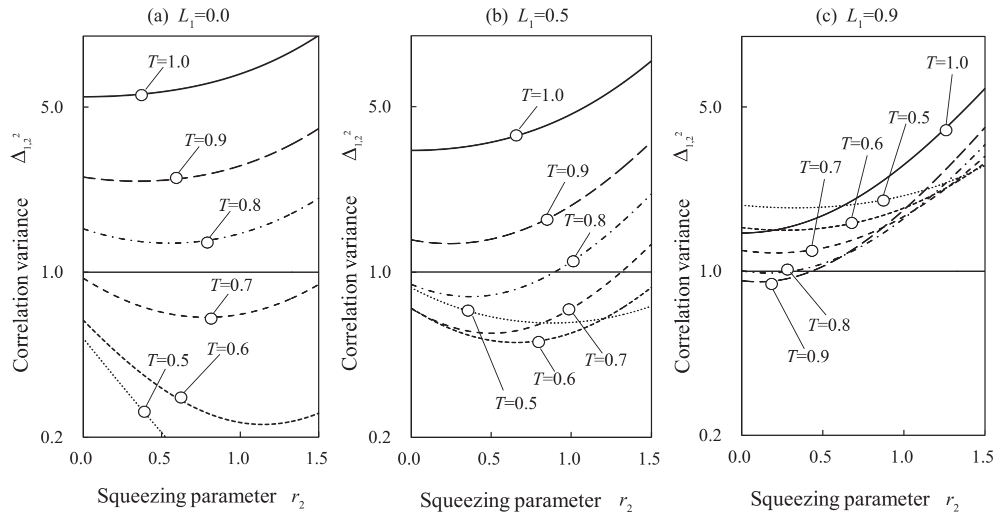

Fig. 2. Calculation results of correlation variance $\Delta_{1,2}{}^2$ in a log scale as a function of squeezing parameter r_2 with beam-splitter transmittance T of 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0, respectively. Squeezing parameter r_1 is fixed at 1.5. Optical loss L_2 is zero and L_1 is set at (a) 0.0, (b) 0.5, and (c) 0.9.

In Fig. 2 (c) with L_2=0.9, most of the correlation variances $\Delta_{1,2}{}^2$ are more than one and don't satisfy the inseparability criterion except those with T=0.8 and 0.9. Correlation variances $\Delta_{1,2}{}^2$ gives the minimum value of 0.901 with $T = 0.9$ at squeezing parameter r_2 of 0.106. At this condition the average photon numbers of incident signal and reference beams n_1, n_2 are estimated as 4.08 and 0.408, respectively. After exposition to the asymmetric optical loss, the average photon number of the final signal beam n_1' is also estimated as 0.464 and becomes comparable to that of reference beam n_2.

Fig. 3 shows calculation results of correlation variance $\Delta_{1,2}{}^2$ as a function of squeezing parameter r_2 with beam-splitter transmittance T of 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0, respectively. Optical loss L_2 was set at zero and L_1 was set at (a) 0.0, (b) 0.5, and (c) 0.9. Squeezing parameter r_1 of incident squeezed vacuum state SQ_1 was fixed at 3.0. Because of this correlation variances $\Delta_{1,2}{}^2$ have high sensitivity to the change of beam-splitter transmittance T as compared with Fig. 2.

Fig. 3 (a) is a result under the lossless condition (L_1=L_2=0) same as Fig. 2 (a). Correlation variances $\Delta_{1,2}{}^2$ monotonically increase when beam splitter transmittance T increases. In Fig. 3 (b) with optical loss L_1 of 0.5, correlation variance $\Delta_{1,2}{}^2$ only with T=0.7 gives the value less than one and has the minimum value of 0.720 at squeezing parameter r_2 of 0.500. In Fig. 3 (c) with L_1=0.9, correlation variances $\Delta_{1,2}{}^2$ only with T=0.9 shows the value less than one and then satisfies the inseparability criterion. It gives the minimum value of 0.922 with $T = 0.9$ at squeezing parameter r_2 of 0.106. At this condition the average photon numbers of incident signal and reference beams n_1, n_2 are estimated as 90.32 and 9.03, respectively. The average photon number of the final signal beam n_1' is also estimated as 10.05 and becomes comparable to that of reference beam n_2 after exposition to the asymmetric optical loss.

As a result of Fig. 2 and 3 the asymmetric property of two-mode squeezed vacuum states has an ability to compensate the effect of asymmetric optical-loss conditions (L_1=0.5, 0.9). We could find out some conditions where the inseparability condition is maintained. And at these conditions the average photon number of final signal beam n_1' and that of reference beam n_2 are almost same.

Fig. 4 shows calculation results of correlation variance $\Delta_{1,2}{}^2$ at the condition with extremely large optical loss L_1 of 0.99 and L_2 of zero. Fig. 4 (a) shows results with squeezing parameter r_1 of 1.5, and beam-splitter transmittance T varying from 0.96, 0.97, 0.98, 0.99, and to 0.995, and (b) shows results with r_1 of 3.0 and T varying from 0.986, 0.988, 0.990, 0.992, and to 0.994. In both cases (a) and (b) correlation variances $\Delta_{1,2}{}^2$ gives the value less than one with T=0.99 and the minimum value of 0.990 at squeezing parameter r_2 of 0.01. At this condition the average photon numbers of signal and reference beams n_1, n_2 are estimated as 4.48 and 0.0448 in (a) and 99.35 and 0.994 in (b), respectively. After exposition to the asymmetric optical loss, the average photon number of final signal beam n_1' is also estimated as 0.0454 in (a) and 1.004 in (b), respectively, and is comparable to that of reference beam n_2.

From these results it is expected that correlation variance $\Delta_{1,2}{}^2$ gives its minimum and the value less than one with high beam-splitter transmittance even under the symmetric optical-loss condition with extremely high

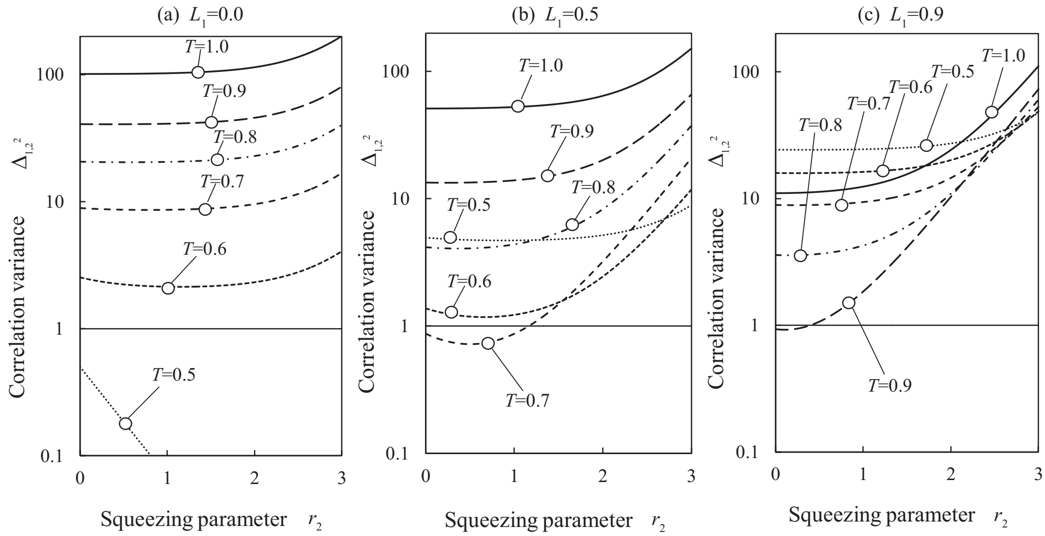

Fig. 3. Calculation results of correlation variance ${\Delta_{1,2}}^2$ in a log scale as a function of squeezing parameter r_2 with transmittance T of beam splitter of 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0, respectively. Squeezing parameter r_1 is fixed at 3.0. Optical loss L_2 is zero and L_1 is set at (a) 0.0, (b) 0.5, and (c) 0.9.

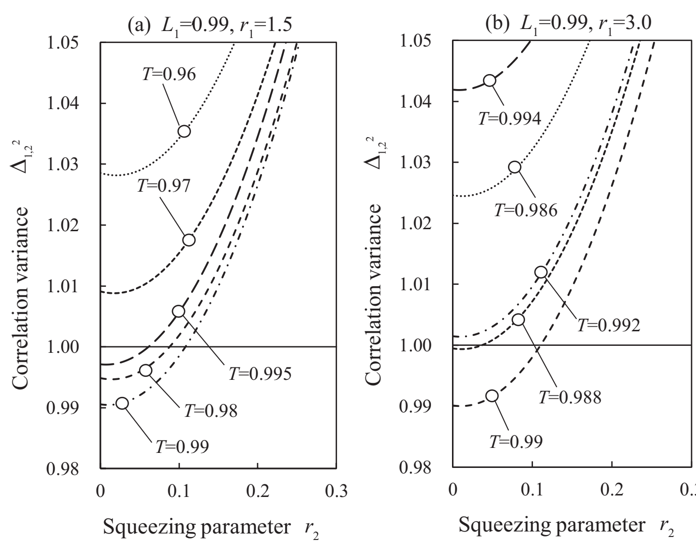

Fig. 4. Calculation results of correlation variance ${\Delta_{1,2}}^2$ in a linear scale as a function of squeezing parameter r_2 at extremely large optical loss L_1 of 0.99. (a) Results with squeezing parameter r_1 of 1.5, and beam splitter transmittance T of 0.96, 0.97, 0.98, 0.99, and 0.995, respectively. (b) Results with squeezing parameter r_1 of 3.0, and beam splitter transmittance T of 0.986, 0.988, 0.990, 0.992, and 0.994, respectively. Optical loss L_2 is set at zero for both cases.

L_1. At this condition squeezing parameter seems to go to almost zero. It means that the incident single-mode vacuum state SQ_2 is almost vacuum state. So, there is a possibility that we can save one single-mode squeezer $SQ_2(r_2)$ in the optical model as shown in Fig. 5. Only one single-mode squeezer $SQ_1(r_1)$ and a beam splitter with high transmittance T are required for generating highly asymmetric two-mode squeezed vacuum states which is efficient light source in asymmetric optical-loss conditions with extremely high L_1.

IV. Summary

The author is interested in applying two-mode squeezed vacuum states to quantum illumination radar. In quantum illumination radar the two-mode entangled light beams are exposed to an asymmetric optical-loss which causes the violation of their inseparability criterion. In this article we assumed asymmetric two-mode squeezed vacuum states. In these states the signal beam has larger quadrature phase amplitudes and an average photon number than the reference beam. We calculated

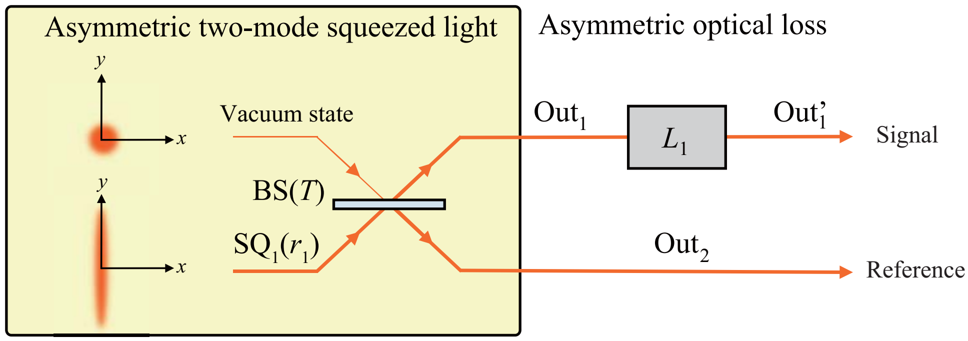

Fig. 5. Optical model for generating asymmetric two-mode squeezed vacuum states which is effective in asymmetric optical-loss conditions with extremely high L_1. Incident single-mode squeezed vacuum state SQ_1 characterized by squeezing parameter r_1 is combined with a vacuum state using beam splitter BS(T) with high transmittance T.

a correlation variance of quadrature phase amplitudes between entangled signal and reference beams after exposition to the asymmetric optical-loss condition. And we studied their non-classical correlation based on Duan's and Simon's inseparability criterion. We could find out some conditions where the asymmetric property of two-mode squeezed vacuum states has an ability to compensate the effect of asymmetric optical-loss conditions and, therefore, can maintain the inseparability condition. Especially in highly asymmetric optical-loss conditions, we could find a possibility that we can save one single-mode squeezer. At this condition only one single-mode squeezer and a beam splitter with high transmittance are required for generating asymmetric two-mode squeezed vacuum states. This is advantageous for developing an entangled light source towards quantum illumination radar.

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