Use of entangled coherent states in quantm teleportation

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I present in this talk mostly the work done at Allahabad in collaboration with Dr. Manoj Kumar Mishra on use of entangled coherent states in almost perfect quantum teleportation and related phenomenon for moderately large coherent amplitudes. It is shown that although entanglement is a necessary resource, in some cases, use of non-maximally entangled states may lead to better minimum average fidelity and hence may be more desirable for small $|\alpha|$, if we use superposition of coherent states with mean photon number $\sim |\alpha|^2$. For such cases a strategy of using one of the two possible unitary transformations for each result of the Bell state measurement is given.

It is shown that a single mode superposed coherent state can encode a qudit of information of any arbitrary dimension, but larger dimensions of qudit may complicate very much the scheme of teleportation. The case of quantum teleportation of a ququat of information is discussed in detail and it is shown that an almost perfect teleportation (fidelity ≥ 0.99) may result if the coherent amplitude $|\alpha| \geq 3.2$.

In usual quantum teleportation schemes, the information is destroyed the moment a Bell state measurement is done. When entangled coherent states are used, certain results of Bell state measurements give perfect failure in the sense that there is no prescription available for making a unitary transformation resulting in teleportation with an acceptable fidelity. This is the reason why quantum teleportation cannot be perfect even in principle using entangled coherent states.

We propose here a scheme for long distance atomic teleportation with perfect fidelity and with success as large as desired using entangled coherent states and cavity assisted interactions. Here photons carry information from one cavity to another and the Bell state measurement is done in two steps. The first step tells whether a success is perceived and does not destroy the information. Thus, one is permitted to start the Bell state measurements afresh if a failure is indicated in the first step. This scheme has the advantage of having deterministic generation of entangled coherent state, robustness of entangled coherent state against decoherence due to absorption and does not require multi-stage cavity interaction or single photon detection ability.

We also study quantum discord of Werner states and quasi-Werner states made using entangled coherent states.

I. INTRODUCTION

I wish to present here mostly the work done in collaboration with Dr. Manoj Kumar Mishra on use of entangled coherent states in quantum teleportation. As all introductory remarks needed here have already been presented by Prof. Ranjana Prakash in her talk, I shall be brief in introduction.

Coherent states are eigenstates of the annihilation operator a, i.e., $a|\alpha\rangle = \alpha|\alpha\rangle$, $|\alpha\rangle = \sum_n \exp(-|\alpha|^2/2)(\alpha^n/\sqrt{n!})|n\rangle$. Commonly considered superposed coherent states (SCS) used as information are single mode states

$$\begin{aligned}|I\rangle &= \epsilon_+|\alpha\rangle + \epsilon_-|-\alpha\rangle \\ &= A_+|\text{EVEN},\alpha\rangle + A_-|\text{ODD},\alpha\rangle,\end{aligned} \tag{1}$$

where

$$\begin{aligned}|\text{EVEN},\alpha\rangle &= (|\alpha\rangle + |-\alpha\rangle)/\sqrt{2(1+x^2)} \\ &= \sum_{n=0}^{\infty} \sqrt{2x/(1+x^2)}\frac{\alpha^{2n}}{\sqrt{2n!}}|2n\rangle, x \equiv e^{-|\alpha|^2}\end{aligned} \tag{2}$$

$$\begin{aligned}|\text{ODD},\alpha\rangle &= (|\alpha\rangle - |-\alpha\rangle)/\sqrt{2(1-x^2)} \\ &= \sum_{n=0}^{\infty} \sqrt{2x/(1-x^2)}\frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}}|2n+1\rangle\end{aligned} \tag{3}$$

are even and odd coherent states. To abbreviate we write these states as $|\pm\rangle$ whenever there is no chance of confusion. The coefficients satisfy

$$\begin{aligned}&\epsilon_\pm = \frac{A_+}{\sqrt{2(1+x^2)}} \pm \frac{A_-}{\sqrt{2(1-x^2)}}, \\ &A_\pm = (\epsilon_+ \pm \epsilon_-)\sqrt{\frac{1 \pm x^4}{2}}, \\ &A_+ = \cos\frac{\theta}{2}, A_- = \sin\frac{\theta}{2}e^{i\phi}, \\ &|A_+|^2 + |A_-|^2 = 1, \\ &|\epsilon_+|^2 + |\epsilon_-|^2 + x^2(\epsilon_+\epsilon_- + \epsilon_-\epsilon_+) = 1.\end{aligned} \tag{4}$$

Since $\langle -\alpha|\alpha\rangle = e^{-2|\alpha|^2} \neq 0$, states $|\pm\alpha\rangle$ are not orthogonal and do not form a useful basis. To cope with this problem, we use the basis of even and odd coherent states.

A state is represented on Bloch sphere by angles (θ, ϕ) defined by $A_+ = \cos\frac{\theta}{2}, A_- = \sin\frac{\theta}{2}e^{i\phi}$. Entanglement and Concurrence have already been introduced by earlier

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speakers, and for pure and mixed states these are [1]

$$C(\psi)_{AB} = |\langle\psi|\sigma_y \otimes \sigma_y|\psi^*\rangle| = |\langle\psi|\tilde{\psi}\rangle|, \quad (5)$$

$$C(\rho_{AB}) = max\,\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \quad (6)$$

It is worthwhile to investigate what happens if a non-maximally state (NME) is taken as resource. The common belief is that it result in a reduction in fidelity, and this happens for the case of atomic qubits. Verma and the speaker [2] investigated this problem in detail for pure non-maximally entangled sates. We found that for a general non-maximally entangled resource information state dependent fidelity is obtained and that the Minimum Assured Fidelity (MASFI) defined a the minimum fidelity for any possible information state is given by MASFI $= (2C/(C+1))$, where C is concurrence.

We also concluded [2] that MASFI is better measure of quality of imperfect teleportation than concurrence or Minimum Average Fidelity.

II. TELEPORTATION OF SUPERPOSED COHERENT STATES USING NON-MAXIMALLY ENTANGLED RESOURCE

van Enk and Hirota [3] showed how to a SCS encoded with one qubit can be teleported using ECS with success probability equal to 0.5. Wang [4] showed how to teleport a bipartite ECS encoded with one qubit using ECS with success probability equal to 0.5. Naresh Chandra, Ranjana Prakash, Shivani and I [5] modified the photon counting scheme and reported almost perfect teleportation for an appreciable mean photon number. Many other schemes proposed the teleportation of SCS using ECS. However most of the schemes used maximally entangled coherent state (MECS) $|E\rangle_{1,2} \sim [|\alpha,\alpha\rangle - |-\alpha,-\alpha\rangle]_{1,2}$ as quantum channel.

We consider more practical problem of teleporting SCS using non-maximally entangled coherent state (NMECS) and study the effect of entanglement on the quality of teleportation [6]. For teleportation we use the bipartite ECS, $|E\rangle_{1,2} = N[\cos\frac{\theta}{2}|\alpha,\alpha\rangle + \sin\frac{\theta}{2}e^{i\phi}|-\alpha,-\alpha\rangle]_{1,2}$, $\theta \in [0,\pi]$, $\phi \in [0,2\pi)$, $N = (1 + x^4 \sin\theta\cos\phi)^{-1/2}$, $x \equiv \exp(-|\alpha|^2)$.

ECS can be expressed in terms of Even and Odd coherent states in the form

$$|E\rangle_{1,2} = \frac{N}{2}[C_+(1+x^2)|+,+\rangle + C_+(1-x^2)|-,-\rangle + C_-\sqrt{(1-x^4)}(|+,-\rangle + |-,+\rangle)], \quad (7)$$

where $C_\pm = \cos\frac{\theta}{2} \pm \sin\frac{\theta}{2}e^{i\phi}$. Concurrence [1] is given by relation, $C = \left|(1-x^4)\sin\theta\cos\phi/\left(1 + x^4\sin\theta\cos\phi\right)\right|$. For $\theta = \pi/2$, $\phi = \pi$, we obtain the MECS $|E_{\theta=\pi/2,\phi=\pi}\rangle_{1,2} = \frac{1}{\sqrt{2}}[|+,-\rangle + |-,+\rangle]_{1,2}$ used in most of the previously proposed schemes for QT of SCS. For $\theta = \pi/2, \phi = 0$ or 2π, $|E_{\theta=\pi/2,\phi=0 \text{ or } 2\pi}\rangle_{1,2} = (2(1+x^4))^{-1/2}[(1+x^2)|+,+\rangle + (1-x^2)|-,-\rangle]_{1,2}$ having $C = (1-x^4)(1+x^4)$. This is an important NMECS. We note here that when $|\alpha|^2 \to \infty, x \to 0$ and concurrence $C \to 1$.

In FIG. 1, '$\oplus$' sign represents the MECS with unit concurrence and two '$\otimes$' signs represents a particular NMECS with concurrence lesser than unit for low value of $|\alpha|^2$.

However, we see that concurrence of particular NMECS becomes almost equal to unity for appreciable value of $|\alpha|^2$. Since for low values of $|\alpha|^2$, ECS is a NMECS except at points represented by '$\otimes$', therefore it will be interesting to study how the quality of teleportation of SCS is affected by the amount of entanglement contained in ECS.

Let Alice desire to teleport information state (SCS) given by Eq. (1), with coefficients satisfying Eqs. (4). Also let the MECS be used as the quantum channel $|E\rangle_{1,2} = N[\cos\frac{\theta}{2}|\alpha,\alpha\rangle + \sin\frac{\theta}{2}e^{i\phi}|-\alpha,-\alpha\rangle]_{1,2}$.

We find the final state in the modes 2, 3, 4, express the coherent states in modes 3 and 4 in terms of zero-photon state (the vacuum state), state with nonzero even numbers of photons and state with odd numbers of photons given by

$$|\pm\sqrt{2}\alpha\rangle = x|0\rangle + \frac{1}{\sqrt{2}}(1-x^2)|\text{NZE},\sqrt{2}\alpha\rangle \pm\sqrt{\frac{1}{2}(1-x^4)}|\text{ODD},\sqrt{2}\alpha\rangle, \quad (8)$$

where, $|\text{NZE},\sqrt{2}\alpha\rangle = (\sqrt{2}(1-x^2))^{-1}[(|\sqrt{2}\alpha\rangle + |-\sqrt{2}\alpha\rangle) - 2x|0\rangle]$, and $|\text{ODD},\sqrt{2}\alpha\rangle = (2(1-x^4))^{-1/2}[|\sqrt{2}\alpha\rangle - |-\sqrt{2}\alpha\rangle]$ and the state with Bob in terms of even and odd coherent states defined by Eqs. (2, 3).

There are five possible photon counting results, (1) zero counts in modes 3 and 4, (2) zero counts in mode 3 and non-zero even in mode 4, (3) nonzero even counts in mode 3 count zero in mode 4, (4) zero count in mode 3 and odd in mode 4, (5) odd counts in mode 3 and zero in mode.

Residual states with Bob in mode 2 after measurement corresponding to each PC results are

$$|B_I\rangle_2 \sim (p^{-1}C_+|+\rangle + q^{-1}C_-|-\rangle)_2$$

$$|B_{II}\rangle_2 \sim [p^{-1}(C_+A_+p + C_-A_-q)|+\rangle + q^{-1}(C_-A_+p + C_+A_-q)|-\rangle]_2$$

$$|B_{III}\rangle_2 \sim [p^{-1}(C_+A_+p - C_-A_-q)|+\rangle + q^{-1}(C_-A_+p - C_+A_-q)|-\rangle]_2$$

$$|B_{IV}\rangle_2 \sim [p^{-1}(C_-A_+p + C_+A_-q)|+\rangle + q^{-1}(C_+A_+p + C_-A_-q)|-\rangle]_2$$

$$|B_V\rangle_2 \sim [p^{-1}(C_-A_+p - C_+A_-q)|+\rangle + q^{-1}(C_+A_+p - C_-A_-q)|-\rangle]_2$$

with $p = \sqrt{1+x^2}$, $q = \sqrt{1-x^2}$ and $C_\pm = \cos\frac{\theta}{2} \pm \sin\frac{\theta}{2}e^{i\phi}$.

Since Bob's state depends on C_+, the strategy is that if $|C_+| \leq |C_-|$ (i.e., $\cos\phi \geq 0$), Bob performs the unitary operations: $U_I = U_{II} = I, U_{III} = |+\rangle\langle+| - |-\rangle\langle-|, U_{IV} = |+\rangle\langle-| + |-\rangle\langle+|, U_V = |+\rangle\langle-| - |-\rangle\langle+|$

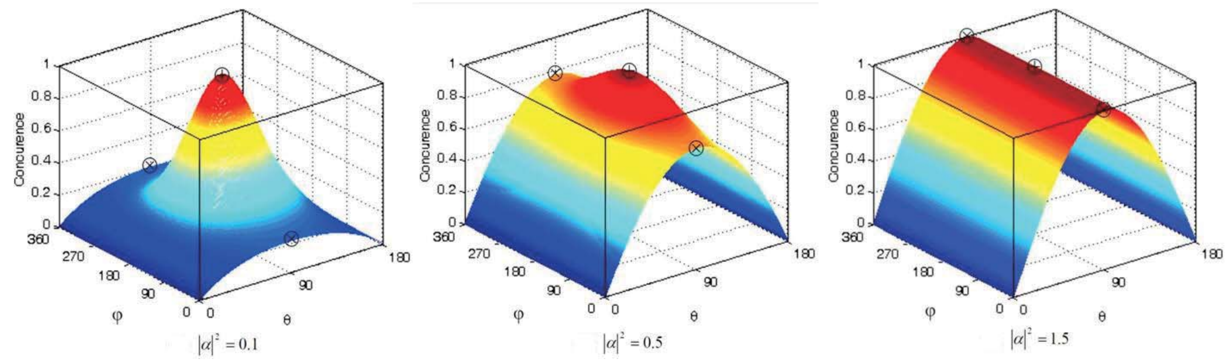

FIG. 1.

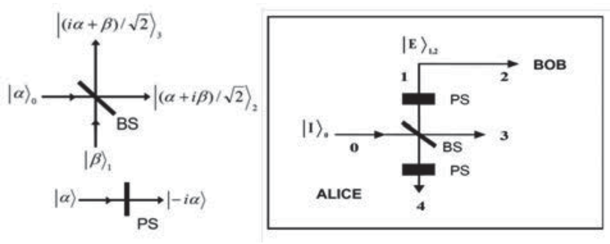

FIG. 2. Upper of the left side figure shows the effect of 50:50 symmetric beam splitter (BS), lower of the left side figure shows $-\pi/2$ phase shifter (PS) on a coherent state and right side figure shows the teleportation scheme.

and if $|C_+| > |C_-|$ (i.e., $\cos\phi < 0$), Bob performs the unitary operations: $U_I = U_{IV} = I, U_{II} = |+\rangle\langle-| + |-\rangle\langle+|, U_{III} = |+\rangle\langle-| - |-\rangle\langle+|, U_V = |+\rangle\langle+| - |-\rangle\langle-|$. Complicated expressions for teleported state, probabilities for outcome of BSM and Fidelities are given in reference [6].

Minimum average fidelity is defined as minimum possible value of average fidelity over all possible information states. We minimized average fidelity, over all possible information states, and then plot it with respect to entanglement parameters θ and ϕ. Also, we take the expressions corresponding to $|C_+| \le |C_-|$ or $|C_+| > |C_-|$. The results are shown in FIG. 3.

For low values of $|\alpha|^2$, the contribution of NMECS $|E_{\theta=\pi/2,\phi=0,2\pi}\rangle_{1,2}$, given by $F^{(1)}_{min.,av} = 1 - \frac{1}{2}x^2(1+x^2)/(1+x^4)$, is higher than the contribution of MECS $|E_{\theta=\pi/2,\phi=\pi}\rangle_{1,2}$, given by $F^{(2)}_{min.,av} = 1 - 2x^2/(1+x^2)^2$. Difference between these two given by $D = x^2(3+x^4)(1-x^2)/2(1+x^4)(1+x^2)^2$ is plotted against $|\alpha|^2$ in FIG. 4.

III. TELEPORTATION OF ONE QUQUAT ENCODED IN SUPERPOSITION OF COHERENT STATE

We now show [7] that encoding of a ququat or, in general, a qudit in a single mode SCS, is possible. We also propose a linear optical scheme that gives almost perfect teleportation (minimum average fidelity > 0.99) of single ququat encoded in single mode SCS with the aid of entangled ququat based on coherent states and using a 9-bit classical channel with almost perfect success rate. We define four multi-photonic states by $|\alpha_0\rangle = N_0[|\alpha\rangle + |i\alpha\rangle + |-\alpha\rangle + |-i\alpha\rangle], |\alpha_1\rangle = N_1[|\alpha\rangle - i|i\alpha\rangle - |-\alpha\rangle + i|-i\alpha\rangle], |\alpha_2\rangle = N_2[|\alpha\rangle - |i\alpha\rangle + |-\alpha\rangle - |-i\alpha\rangle]$ and $|\alpha_3\rangle = N_3[|\alpha\rangle + i|i\alpha\rangle - |-\alpha\rangle - i|-i\alpha\rangle]$. These states are orthonormal, $\langle\alpha_j|\alpha_k\rangle = \delta_{jk}$, and complete, and contain 4n, 4n+1, 4n+2, and 4n+3 photons respectively. Normalization constants are given by $N_{0,2} = [2(1+x^2 \pm 2x\cos|\alpha|^2)^{1/2}]^{-1}$, and $N_{1,3} = [2(1-x^2 \pm 2x\sin|\alpha|^2)^{1/2}]^{-1}$. We can solve these equations and get $|\pm\alpha\rangle = \frac{1}{2}[r_0|\alpha_0\rangle \pm r_1|\alpha_1\rangle + r_2|\alpha_2\rangle \pm r_3|\alpha_3\rangle], |\pm i\alpha\rangle = \frac{1}{2}[r_0|\alpha_0\rangle \pm ir_1|\alpha_1\rangle - r_2|\alpha_2\rangle \mp ir_3|\alpha_3\rangle]$, where $r_j = (2N_j)^{-1}$. We define four entangled ququat states based on coherent state, say, four bipartite four-component entangled coherent states (BFECS) as

$$|E_{0,2}\rangle = N_{E_0,E_2}[|\alpha,\alpha\rangle \pm |i\alpha,i\alpha\rangle + |-\alpha,-\alpha\rangle \pm |-i\alpha,-i\alpha\rangle] \quad (9)$$

$$|E_{1,3}\rangle = N_{E_1,E_3}[|\alpha,\alpha\rangle \mp i|i\alpha,i\alpha\rangle - |-\alpha,-\alpha\rangle \pm i|-i\alpha,-i\alpha\rangle] \quad (10)$$

where $N_{E_0,E_2} = [2(1+x^2 \pm 2x\cos|\alpha|^2)^{1/2}]^{-1}$, $N_{E_1,E_3} = [2(1-x^2 \pm 2x\sin|\alpha|^2)^{1/2}]^{-1}$ are normalization constants. In terms of orthogonal states $|\alpha_j\rangle$, these BFECS can also be written as $|E_0\rangle = N_{E_0}[r_0^2|\alpha_0,\alpha_0\rangle + r_1r_3|\alpha_1,\alpha_3\rangle + r_2^2|\alpha_2,\alpha_2\rangle + r_3r_1|\alpha_3,\alpha_1\rangle]$ (etc) and are NMECS. For appreciably large coherent amplitude, however, i.e., in the limit, $|\alpha| \to \infty$, the coefficients N_{E_j} and r_j, become almost equal to 1/2 and unity, respectively, and therefore BFECS becomes maximally entangled.

The BFECS can be generated by having the two inputs modes 1 & 2 (see FIG. 5) as even coherent states which give the two mode (4 & 5) output as $|\psi\rangle_{4,5} = N_e^2[|\alpha,i\alpha\rangle + |i\alpha,\alpha\rangle + |-i\alpha,-\alpha\rangle + |-\alpha,-i\alpha\rangle], N_e[2(1+x^2)]^{1/2}$. Photon counting in mode 4 gives results 4n, 4n+1, 4n+2 or 4n+3 and correspondingly at Mode 5 the output is $|\alpha_0\rangle, |\alpha_1\rangle, |\alpha_2\rangle$ or $|\alpha_3\rangle$ respectively. Probability of generation of each state $|\alpha_j\rangle$ is $P_j = N_e^4 r_j^4$, which becomes equal to 0.25 for appreciable value of coherent amplitude

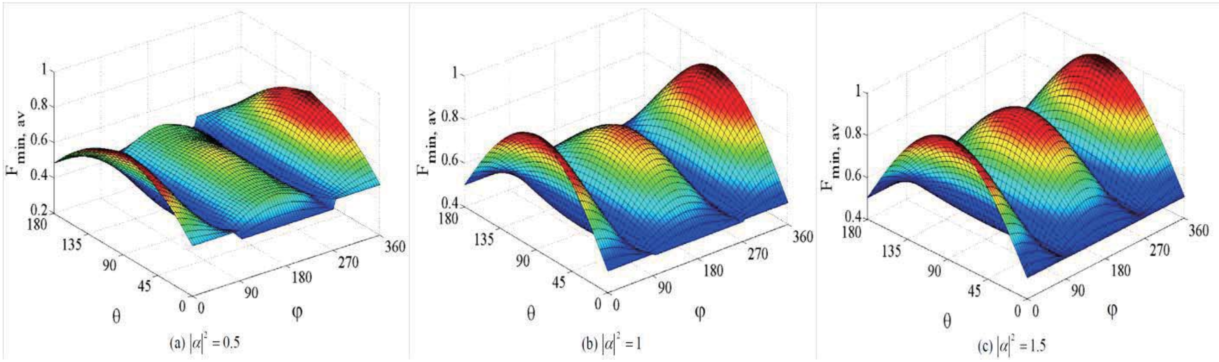

FIG. 3.

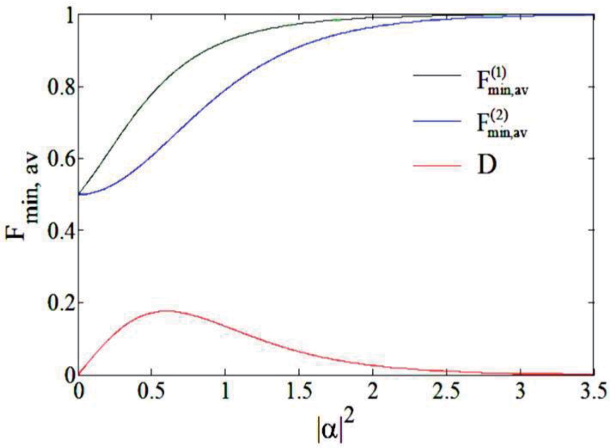

FIG. 4. Variation of $F^{(1)}_{min.,av}$ for non-maximally ECS, $F^{(2)}_{min.,av}$ for maximally ECS and their difference D with respect to mean photon number $|\alpha|^2$. The maximum difference is ≈ 0.17 at $|\alpha|^2 \approx 0.6$.

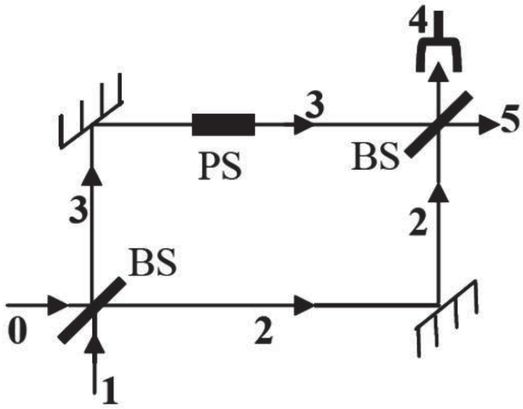

FIG. 5. Scheme for generating BFCECS.

$|\alpha|$. After illuminating a 50-50 BS by state $|\alpha_j\rangle$, the resulting state is an entangled ququat similar to BFECS $|E_j\rangle$.

One ququat information state can be written as

$$\begin{aligned} |I\rangle_1 &= [\epsilon_0|\alpha\rangle + \epsilon_1|i\alpha\rangle + \epsilon_2|-\alpha\rangle + \epsilon_3|-i\alpha\rangle]_1 \\ &= [c|\alpha_0\rangle + c_1|\alpha_1\rangle + c_2|\alpha_2\rangle + c_3|\alpha_3\rangle]_1 \end{aligned} \tag{11}$$

where the coefficients satisfy normalization conditions and $c_{0,2} = \frac{1}{2}r_{0,2}(\epsilon_0 \pm \epsilon_1 + \epsilon_2 \pm \epsilon_3), c_{1,3} = \frac{1}{2}r_{1,3}(\epsilon_0 \pm i\epsilon_1 - \epsilon_2 \mp i\epsilon_3)$. For BFECS quantum channel $|E_0\rangle$, calculations done on the usual line give the Alice-Bob system final state as

$$\begin{aligned} &|\psi\rangle_{1,2,3} = N_{E_0}[\epsilon_0(|i\alpha, 0, \beta, \beta, \alpha\rangle + |i\beta, -i\beta, 0, i\alpha, i\alpha\rangle \\ &+|0, \alpha, -i\beta, i\beta, -\alpha\rangle + |\beta, \beta, \alpha, 0, -i\alpha\rangle) \\ &+\epsilon_1(|i\beta, i\beta, i\alpha, 0, \alpha\rangle + |-\alpha, 0, i\beta, i\beta, i\alpha\rangle \\ &+|-\beta, \beta, 0, -\alpha, -\alpha\rangle + |0, i\alpha, \beta, -\beta, -i\alpha\rangle) \\ &+\epsilon_2(|0, -\alpha, i\beta, -i\beta, \alpha\rangle + |-\beta, -\beta, -\alpha, 0, i\alpha\rangle \\ &+|-i\alpha, 0, -\beta, -\beta, -\alpha\rangle + |-i\beta, i\beta, 0, -i\alpha, -i\alpha\rangle) \\ &+\epsilon_3(|\beta, -\beta, 0, \alpha, \alpha\rangle + |0, -i\alpha, -\beta, \beta, i\alpha\rangle \\ &+|-i\beta, -i\beta, -i\alpha, 0, -\alpha\rangle \\ &+|\alpha, 0, -i\beta, -i\beta, -i\alpha\rangle)]_{8,9,10,11,3} \end{aligned}$$

with $|\beta\rangle = |\frac{1}{2}(1+i)\alpha\rangle$. Alice performs the photon counting (PC) in modes 8, 9, 10, and 11 and conveys her PC result to Bob, on the basis of which Bob performs an appropriate unitary operation on his mode 3 to get faithful replica of the original information state. By writing

$$\begin{aligned} |\pm\alpha\rangle &= a_0|0\rangle \pm a_1|\alpha_1\rangle + a_2|\alpha_1\rangle \pm a_3|\alpha_3\rangle + a_4|\alpha_4\rangle \\ |\pm i\alpha\rangle &= a_0|0\rangle \pm ia_1|\alpha_1\rangle - a_2|\alpha_1\rangle \mp ia_3|\alpha_3\rangle + a_4|\alpha_4\rangle \end{aligned}$$

where $a_0 = \sqrt{x}, a_{1,2,3} = \frac{1}{2}r_{1,2,3}, a_4 = \frac{1}{2}\sqrt{r_0^2 - 4x}$ and similar relations for $|\pm\beta\rangle$ and $|\pm i\beta\rangle$ with expressions for b_j obtainable from those of a_j, by replacing α_j by β_j.

Using these expansions one can verify that one of the modes 8, 9, 10, and 11 always has vacuum state and each of the other three modes can give any of the five results, zero or nonzero, which is 0, 1, 2 or 3 (modulo 4).

Thus, there are $C^4_1 4^3 + C^4_2 4^2 + C^4_3 4^1 + C^4_4 4^0 = 369$ different PC results. These results can be transmitted to Bob on a 9-bit classical channel. Since Bob has to know only the required unitary transformation and there are only 64 distinct unitary transformations, even 8 c-bit channel is sufficient. We write these PC results as 0, 1, 2, 3 and 4, the last one being the nonzero result (0 modulo 4) written as 4 to distinguish it from the result of 0 counts and classify them into four groups:

Group I (All modes count zero photon),

Group II (Any three modes count zero and one mode count non-zero photon),

Group III (Any two modes count zero photon and rest two modes count non-zero photon), and

Group IV (Only one mode count zero and rest three modes count non-zero photons).

In Group I, there is only one case of result and the teleported state is seen to be $\sim |\alpha\rangle + |i\alpha\rangle + |-\alpha\rangle + |-i\alpha\rangle$ irrespective of the information. If the information is in this state F = 1 and if the information is orthogonal to it F = 0. Thus MASFI is 0 and we say that the **Teleportation Fails. This case is however important for small $|\alpha|^2$, and for $|\alpha|^2 > 1.5$ probability for occurrence of this case is nearly zero**.

In Group II, there are 16 possible PC results as the non-zero photon mode may be any one of the four modes and non-zero photon counts may be any of 4n+1, 4n+2, 4n+3 or 4n+4. Since nonzero counts may be obtained both for $|\alpha_4\rangle$ and for $|\beta_4\rangle$, one cannot devise a prescription for the required unitary transformation to be performed by Bob and hence **Teleportation Fails**.

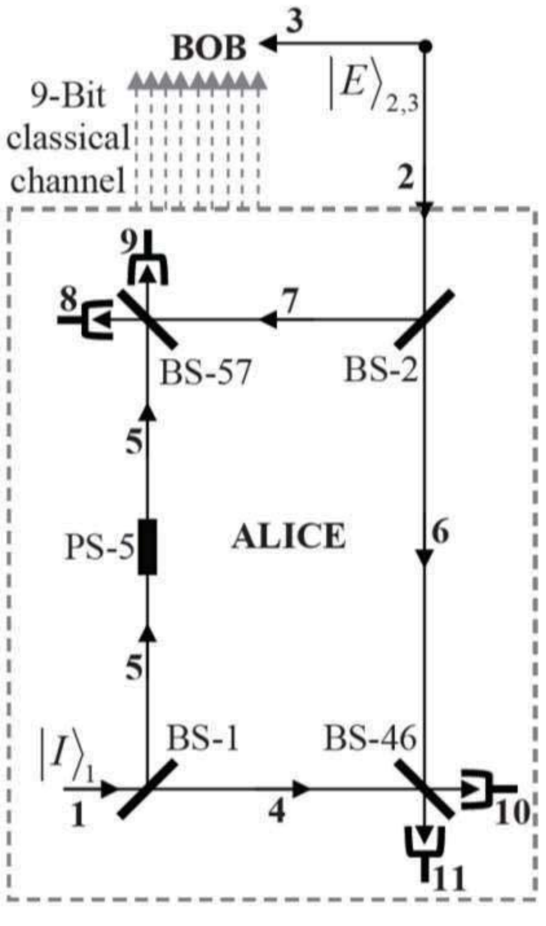

FIG. 6. Scheme for teleporting one ququat encoded in superposition of coherent states with the aid of entangled ququat based on coherent state called BFECS. BS and PS stands for 50-50 beam splitter and $-\pi/2$ phase shifter, respectively. Bold numbers represent the quantum mode.

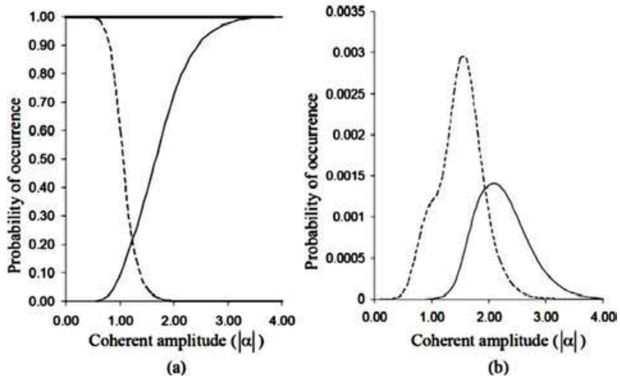

FIG. 7. (a) Dashed curve shows variation of maximum probability of occurrence for photon counting result (0000) of group I, with respect to coherent amplitude. Continuous curve shows the variation of summation of probabilities of occurrence for all 256 photon counting results belonging to Group IV. (b) Dashed and continuous curves shows variations of maximum probability of occurrence with respect to coherent amplitude for typical cases of group II and III of the photon counting result (4000) and (4040) respectively.

In Group III, there are $C_2^4 4^2 = 96$ PC results, which may further be divided into two subgroups, Subgroup III.I and Subgroup III.II.

Subgroup III.I (Pair of modes '8 and 10' or '8 and 11' or '9 and 10' or '9 and 11' show zero counts, while the rest two modes show non-zero photons): This subgroup has $C_1^4 4^2 = 64$ PC results. The situation for this case is exactly similar to that discussed for Group II and **Teleportation Fails**.

FIG. 7 shows variation of maximum probability of occurrence for PC result of different groups. From where it is clear that probability of occurrence for PC result belonging to groups I, II and III becomes zero for appreciable coherent amplitude. **Thus Occurrence of PC results belonging to Groups I, II, III will not degrade the average fidelity for $|\alpha| \geq 3.2$.**

Subgroup III.II (modes '8 and 9' or '10 and 11' counts zero, while rest mode count non-zero photons): This subgroup has 32 PC results. If we look at the states with Bob for the 32 PC results, it is seen that the Bob's state is invariably in the form $B^{(j,k,m)} = \frac{1}{2}[B^{(j,k)} + i^m B^{(j+2,k)}]$, where $B^{(j,k)} = \sum_{l=0}^{3} c_{l+k}(r_l/r_{l+k}) i^{jl}|\alpha_l\rangle$. For 16 cases, a unitary transformation resulting in perfect or almost perfect teleportation exists, the required unitary transformations for the Bob's state $B^{(j,k,m)}$ is $U^{(j,k,m)} = \frac{1}{2}[U^{(j,k)} + (-i)^m U^{(j+2,k)}]$, where $U^{(j,k)} = \sum_{l=0}^{3}(-i)^{jl}|\alpha_{k+l}\rangle\langle\alpha_l|$. For the cases where no unitary transformation giving F = 1 is possible and MASFI = 0, we admit failure, but prescribe unitary transforma-

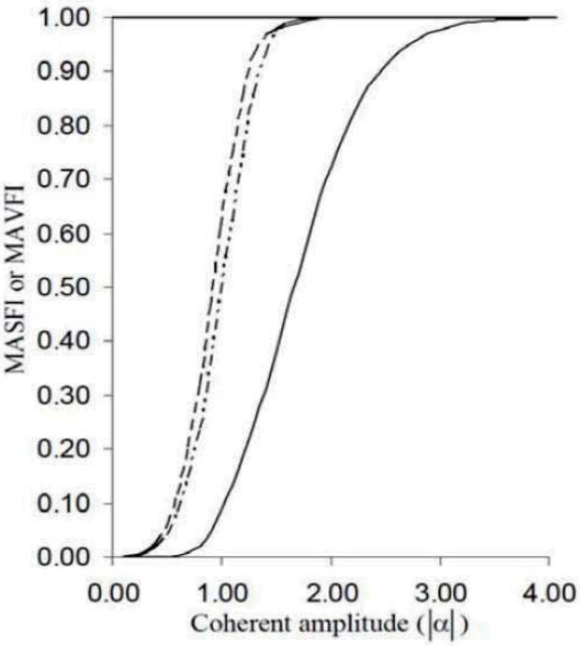

FIG. 8. Dashed curve shows variation of $F_{5,6,8,10}^{\mathrm{MASFI}}$, dash-dotted curve shows variation of F_7^{MASFI} and solid curve shows variation of MAVFI.

tions $U^{j,k}$ which give F = 1 for certain cases of special information states, although MASFI = 0. There are 16 such cases. Table given in Reference [7] shows all 32 PC results belonging to subgroup III.II, corresponding state with Bob, the unitary operations, teleported state and the fidelities in six different forms $F_1, F_2, ..., F_6$. For 16 cases for which fidelity is F5 or F6 [7], MASFI ≈ 1 for $|\alpha| \geq 1.7$. For other 16, MASFI = 0.

FIG. 8 shows that plot of $F_{5,6}^{\mathrm{MASFI}}$ and shows that it reaches unity for $|\alpha| \geq 1.7$. Thus 16 PC results of this subgroup fails to teleport the information state, while rest 16 PC results gives almost perfect teleportation for $|\alpha| \geq 1.7$.

In Group IV there are $C_1^4 4^3 = 256$ PC results. For this group of PC results the Bob's state and unitary transformation are seen to occur in the form, $B^{(j,k)} = \sum_{l=0}^{3} c_{l+k}(r_l/r_{l+k}) i^{jl} |\alpha_l\rangle$ and $U^{(j,k)} = \sum_{l=0}^{3} (-i)^{jl} |\alpha_{k+l}\rangle\langle\alpha_l|$, respectively [7].

For all 256 PC results corresponding Bob's state and required unitary transformation, are tabulated in Reference [7]. PC results for k = 0, 1, 2, 3 lead to fidelities F_7, F_8, F_9 and F_{10}, respectively with $F_7 = 1$, $F_8 = F_5$, $F_{10} = F_6$ [7]. Variation of MASFI against coherent amplitude is shown by dashed curve for k = 1 or 3 and by dash-dotted curve for k = 2 in FIG. 8.

Thus, out of all 256 PC results belonging to Group IV, 64 PC results gives perfect teleportation for any value of $|\alpha|$, while rest 192 PC results gives almost perfect teleportation for $|\alpha| \geq 1.7$. Continuous curve shows minimum average fidelity (MAVFI).

It is clear that $F_{avmin} \geq 0.99$ for $|\alpha| \geq 3.2$ and **thus almost perfect teleportation with perfect success rate is achieved for** $|\alpha| \geq 3.2$.

IV. LONG DISTANCE ATOMIC TELEPORTATION USING ENTANGLED COHERENT STATES AND CAVITY ASSISTED INTERACTION

Large numbers of schemes for teleportation of qubits based on single photon and superposed coherent states (SCS) have been proposed. However, single-photon or SCS are not ideal for long term storage of quantum information as they are very difficult to keep in a certain place. On the other hand, it has been demonstrated that a single atom can be trapped for a few seconds inside an optical cavity. Thus, atoms are ideal for quantum information storage. Numbers of schemes for atomic teleportation using atom-cavity interactions and atoms as flying qubit have been proposed. Since atoms move slowly and interact strongly with their environment, these schemes are unable to perform long distance atomic teleportation and hence can not be used as link between two quantum processors working distant apart. Long distance teleportation is of particular importance because of its applicability in secure quantum communication and future satellite based quantum communication. S Bose et al [8], have presented a novel scheme for teleporting quantum state of an atom trapped in an optical-cavity to second atom in another distant optical-cavity. This scheme involves mapping of atomic state to a cavity state with Alice, followed by the detection of photons leaking out from Alice's cavity and Bob's cavity (initially in maximally entangled atom-cavity state) by mixing over a beam splitter. The main shortcoming of this scheme is that the teleportation fidelity and success rate in this depends on the state to be teleported. Under reasonable cavity parameters and cavity decay time, success rate is near 1/2.

Further Chimczak [9] pointed out the inefficiency of scheme proposed by Bose due to large damping values of currently available cavities that reduces the fidelity of state mapping from atom to cavity and discussed a modification using non-maximally entangled atom-cavity state with amplitudes chosen in such a way that compensates the damping factors due to state mapping. Although this resolves the effect of damping but gives very low success rate. In case of failure, in both schemes the message state is destroyed. Moreover, both schemes are expected to suffer decoherence due to photon absorption while propagating toward beam splitter.

For all these reasons, a dream scheme for long distance atomic teleportation is required that, (i) gives state independent teleportation fidelity and (ii) high success rate, (iii) conserves message state on failure thus permitting repeated attempts and (iv) does not need efficient single photon detection ability, and (v) many matter-light interaction stages. Along with these requirements, the

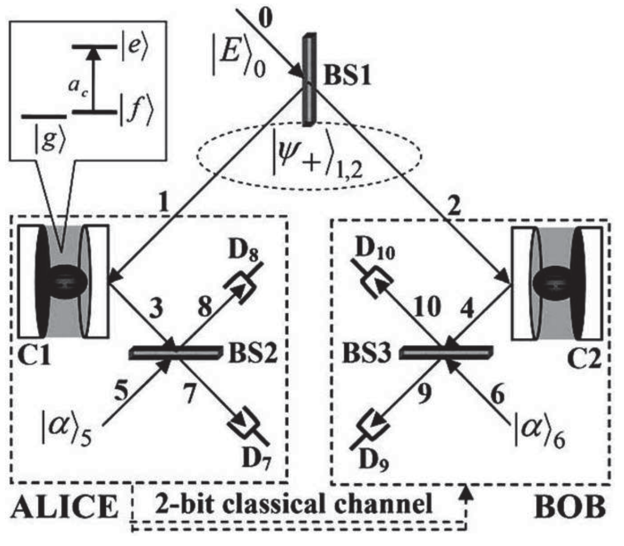

FIG. 9. Scheme for teleportation of atomic-state trapped in cavity C1 to second atom in a distant cavity C2. Entangled coherent state ($|\psi_+\rangle_{1,2} = N_+[|\alpha, \alpha\rangle + |-\alpha, -\alpha\rangle]_{1,2}$) in modes 1 and 2 is produced by illuminating beam splitter BS1 with an even-coherent state ($|E\rangle_0 = N_+[|\sqrt{2}\alpha\rangle + |-\sqrt{2}\alpha 2\rangle]_0$) in mode 0. Inset shows level structure of atom. D1, 2, 3, 4 are photon detectors, atom in cavity C1 is measured in basis ($|\pm\rangle$). Encircled numbers represent the quantum mode.

scheme should use (i) quantum channel that can be deterministically prepared and (ii) must be robust against photon absorption.

Since ECS are more robust against decoherence due to photon absorption than the SBBS [10] and trapped atom in an optical cavity are ideal for quantum information storage, we propose [11] here a scheme for long distance atomic teleportation using ECS that fulfills most of the requirements mentioned above.

Wang and Duan [12] showed that if $|g\rangle$ and $|f\rangle$ are the ground levels with different hyperfine spins and $|e\rangle$ is the excited level, the transition $|f\rangle \rightarrow |e\rangle$ is resonantly coupled to the cavity mode a_c, which is resonantly driven by an input coherent pulse $|\alpha\rangle$. The transition $|g\rangle \rightarrow |e\rangle$ is decoupled to the cavity mode a_c due to large detuning from the hyperfine frequency.

If initial joint state of atom and input pulse is $|g, \pm\alpha\rangle_{c,in}$, then input pulse is resonant with cavity and exact quantum optics calculation by [13], shows that input pulse reflects with a phase change by π, or stating mathematically $|g, \pm\alpha\rangle_{c,in} \rightarrow |g, \mp\alpha\rangle_{c,out}$. For sate $|f, \pm\alpha\rangle_{c,in}$, however, since cavity mode is significantly detuned from the center frequency of the input pulse due to strong atom cavity coupling, $|f, \pm\alpha\rangle_{c,in} \rightarrow |f, \pm\alpha\rangle_{c,out}$.

If Alice desires to teleport message state of an atom in cavity C1 given by $|M\rangle_{C1} = [a|g\rangle + b|f\rangle]_{C1}, |a|^2 + |b|^2 = 1$ to a second atom in a distant cavity C2, initially in state, $|+\rangle_{C2} = \frac{1}{\sqrt{2}}[|g\rangle + |f\rangle]_{C2}$, and they share an entangled coherent state, $|\psi_+\rangle_{1,2} = N_+[|\alpha, \alpha\rangle + |-\alpha, -\alpha\rangle]_{1,2}$, which can be prepared by illuminating a 50:50 beam splitter by even coherent state $|E\rangle_0 = N_+[|\sqrt{2}\alpha\rangle + |-\sqrt{2}\alpha 2\rangle]_0$, calculations on usual lines show that the initial state of the system changes from $|\phi\rangle_{1,2,C1,C2} = |\psi_+\rangle_{1,2}|M\rangle_{C1}|+\rangle_{C2}$ to $|\phi\rangle_{1,4,C1,C2}$

$$\begin{aligned}
&\frac{1}{2}[(|\sqrt{2}\alpha, 0, \sqrt{2}\alpha, 0\rangle + |0, -\sqrt{2}\alpha, 0, -\sqrt{2}\alpha\rangle)_{7,8,9,10} \\
&\times(|+\rangle_{C1}(|M\rangle)_{C2} + |-\rangle_{C1}(\sigma_z|M\rangle)_{C2}) \\
&+(|\sqrt{2}\alpha, 0, 0, -\sqrt{2}\alpha\rangle + |0, -\sqrt{2}\alpha, \sqrt{2}\alpha, 0\rangle)_{7,8,9,10} \\
&\times(|+\rangle_{C1}(\sigma_x|M\rangle)_{C2} + |-\rangle_{C1}(-i\sigma_y|M\rangle)_{C2})]. \qquad (12)
\end{aligned}$$

Now Alice performs photon counting in mode 7 & 8, and performs atomic measurement in diagonal basis in cavity C1, while Bob performs PC in modes 9 & 10. It is clear from the above given output state that two modes always gives zero count. For appreciable value of mean photon numbers of the order of $|\alpha|^2$ all possible measurement results are different and hence appropriate unitary operation can be prescribed to generate exact replica of original information state in cavity C2.

However, since coherent states are the superposition of vacuum state and all photon number state, thus there is nonzero probability to detect vacuum state even when light is present. This results to some nonzero probability of failure at small mean photon numbers $|\alpha|^2$.

To estimate success rate and resolve the problem of failure at small values of $|\alpha|^2$, we expand coherent state $|\pm\sqrt{2}\alpha\rangle$ into vacuum state ($|0\rangle$) and state with nonzero numbers of photons ($|\mathrm{NZ}_\pm\rangle$) given by $\pm\sqrt{2}\alpha\rangle = x|0\rangle + \sqrt{1-x^2}|\mathrm{NZ}_\pm\rangle$. Using this the final output state becomes $|\phi\rangle_{7,8,9,10,C1,C2}$

$$\begin{aligned}
&N_+[|M\rangle_{C1}|+\rangle_{C2}(2x^2|0,0,0,0\rangle \\
&+x\sqrt{1-x^2}|0,0,\mathrm{NZ}_+,0\rangle + |\mathrm{NZ}_+,0,0,0\rangle \\
&+|0,0,0,\mathrm{NZ}_-\rangle + |0,\mathrm{NZ}_-,0,0\rangle)_{7,8,9,10} \\
&+\frac{1}{2}(1-x^2)(|+\rangle_{C1}|M\rangle_{C2} + |-\rangle_{C1}\sigma_z|M\rangle_{C2}) \\
&\times(|\mathrm{NZ}_+,0,\mathrm{NZ}_+,0\rangle + |0,\mathrm{NZ}_-,0,\mathrm{NZ}_-\rangle)_{7,8,9,10} \\
&+\frac{1}{2}(1-x^2)(|+\rangle_{C1}\sigma_x|M\rangle_{C2} + |-\rangle_{C1}(-i\sigma_y)|M\rangle_{C2}) \\
&\times(|\mathrm{NZ}_+,0,0,\mathrm{NZ}_-\rangle + |0,\mathrm{NZ}_-,\mathrm{NZ}_+,0\rangle)_{7,8,9,10}].
\end{aligned}$$

It is clear that two modes of the 7, 8, 9, and 10 are always in vacuum state and measurement results can be classified into two groups:

Group I: Two field modes among 7-10 gives non-zero photon counts and atom in cavity C1 is detected in either of the states $|+\rangle$ or $|-\rangle$.

Group II: Three or all field modes among 7-10 are detected as OFF and atom in cavity C1 is detected in either of the states $|+\rangle$ or $|-\rangle$.

When measurement results falls into group I, Bob's atom can be transformed to the original message state just by applying an appropriate unitary operation given in the Table I against measurement results for successful teleportation. Tick stands for detection of non-zero

TABLE I. Measurement results for successful teleportation.

Alice			Bob		Bob's	Unitary
D_7	D_8	$D_\pm$	D_9	D_{10}	Atomic States	Operation
√	×	+	√	×	M	I
√	×	-	√	×	$\sigma_z M$	σ_z
×	√	+	×	√	M	I
×	√	-	×	√	$\sigma_z M$	σ_z
√	×	+	×	√	$\sigma_x M$	σ_x
√	×	-	×	√	$-i\sigma_y M$	$i\sigma_y$
×	√	+	√	×	$\sigma_x M$	σ_x
×	√	-	√	×	$-i\sigma_y M$	$i\sigma_y$

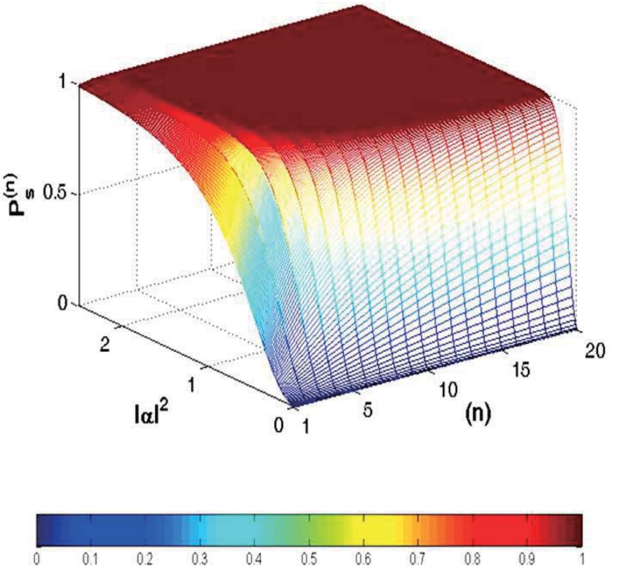

FIG. 10. Shows variation of the success probability ($P_S^{(n)}$) for different numbers of attempts n with $|\alpha|^2$.

photon and cross stands for detection of vacuum by detectors. $\pm$ stands for atomic state in basis $|\pm\rangle$ and σ's are Pauli matrices.

Group I gives perfect teleportation with unit fidelity. The probability of successful teleportation P_s is given by summing the probability of occurrence of all measurement results corresponding to group I, and it is given by relation, $P_s = (1-x^2)^2(1+x^4)^{-1}$.

However, for the measurement results corresponding to group II, teleportation fails. Probability of failure P_f is given by $P_f = 2x^2(1+x^4)^{-1} = 1 - P_s$. But, it is clear that in such case before measurement on atom in cavity C1, the joint state of atoms in cavity C1 and C2 is given by $|M\rangle_{C1}|+\rangle_{C2}$. Thus message state of atoms in cavity C1 and initial state of the atom in cavity C2 remains conserved up to this stage and it enables a fresh attempt.

FIG. 10 shows variation of the success probability ($P_S^{(n)}$) for different numbers of attempts 'n' with $|\alpha|^2$. We plot $P_S^{(n)}$ with respect to $|\alpha|^2$ and 'n' in FIG. 10. In a single attempt 'n = 1', the probability of success increases as $|\alpha|^2$ increases and becomes almost equal to unity for $|\alpha|^2 \geq 2.5$. This is due to the fact that for higher $|\alpha|^2$, probability of detecting vacuum in coherent state becomes almost zero. However for small $|\alpha|^2$, probability of success is appreciably less then unity but increases rapidly with increasing number of attempts 'n'. For example, at $|\alpha|^2 = 1$, success is 0.734, 0.963 or 0.998 for one, two or three attempts.

Thus unit success can be obtained in a single attempt for $|\alpha|^2 \geq 2.5$ or in finite number of attempts for low value of $|\alpha|^2 < 2.5$. We made calculations using a shared non-maximally entangled coherent state $|\psi_+\rangle_{1,2} = N_+[|\alpha,\alpha\rangle + |-\alpha,-\alpha\rangle]_{1,2}$. If we use the maximally entangled coherent state $|\psi_+\rangle_{1,2} = N_+[|\alpha,\alpha\rangle - |-\alpha,-\alpha\rangle]_{1,2}$, it is seen that the success rate increases by a multiplicative factor of $(1+x^2)(1-x^2)$. For this, however, in case of a failure, the message states alter trivially and require transformation by Pauli matrix σ_z for restoration.

V. QUANTUM DISCORD AND ENTANGLEMENT OF QUASI-WERNER STATES BASED ON BIPARTITE ENTANGLED COHERENT STATES

For random variables X and Y, classical mutual information can be written in two equivalent expressions, $I(X:Y) = H(X) + H(Y) - H(X,Y)$ and $J(X:Y) = H(X) - H(X|Y)$. Oliver and Zurek [14] showed that the quantum equivalents of I and J are different and defined quantum discord as the difference,

$$\begin{aligned} D(X:Y)_{\{\prod_j^Y\}} &= I(X:Y) - J(X:Y)_{\{\prod_j^Y\}} \\ &= S(\rho_Y) - S(\rho_{X,Y}) + S(\rho_{X|\{\prod_j^Y\}}). \end{aligned} \quad (13)$$

Here, $\{\prod_j^Y\}$ is a complete set of one-dimensional projector satisfying $\sum_j \prod_j^Y = 1$ and $\rho_{X|\prod_j^Y}$ is the reduced density operator for X when measurement on Y has been done and state j has been detected and is given by $\rho_{X|\prod_j^Y} = (\prod_j^Y \rho_{X,Y} \prod_j^Y)/Tr(\prod_j^Y \rho_{X,Y})$. Interest is also on minimum of this over the set $\{\prod_j^Y\}$, i.e., $\delta(X:Y)_{\{\prod_j^Y\}} = min_{\{\prod_j^Y\}}[D(X:Y)_{\{\prod_j^Y\}}]$.

Investigating QD in some systems is of important significance. On the one hand, it allows us to discover relevant quantum properties of systems. On the other hand, studying QD in physical systems helps and prompts us to explore the theory of QD. Most of the studies related to quantum discord remained focused on qubit systems.

We [15] considered Werner-Like states formed by MECS's $|\psi^\pm\rangle_{XY}$ and NMECS $|\phi^\pm\rangle_{XY}$, which are defined by

$$|\psi^\pm\rangle_{XY} = n_\pm[|\alpha,\alpha\rangle \pm |-\alpha,-\alpha\rangle]_{XY}, \quad (14)$$

$$|\phi^\pm\rangle_{XY} = n_\pm[|\alpha,-\alpha\rangle \pm |-\alpha,\alpha\rangle]_{XY}. \quad (15)$$

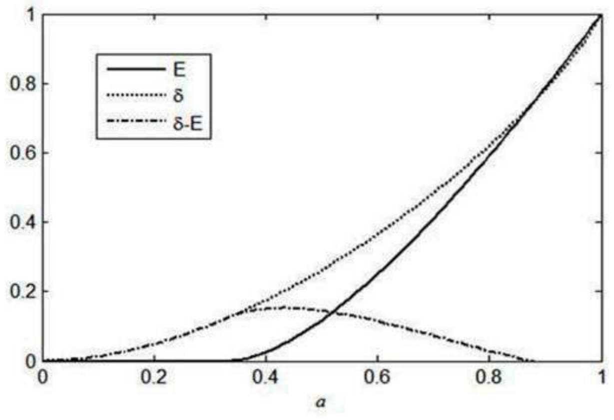

FIG. 11.

The density operators are given by

$$\rho(\psi^+, a) = \frac{1}{4}(1-a)I + a|\psi^+\rangle\langle\psi^+|, \quad (16)$$

$$\rho(\psi^-, a) = \frac{1}{4}(1-a)I + a|\psi^-\rangle\langle\psi^-|, \quad (17)$$

$$\rho(\phi^+, a) = \frac{1}{4}(1-a)I + a|\phi^+\rangle\langle\phi^+|, \quad (18)$$

$$\rho(\phi^-, a) = \frac{1}{4}(1-a)I + a|\phi^-\rangle\langle\phi^-|, \quad (19)$$

For measurement basis, $|\pi_0\rangle = \cos\theta|+\rangle + e^{i\phi}\sin\theta|-\rangle, |\pi_1\rangle = \sin\theta|+\rangle - e^{i\phi}\cos\theta|-\rangle$, we find that the quantum discords for states given by the density operators $\rho(\psi^-, a)$ and $\rho(\phi^-, a)$ do not depend on angles θ, ϕ or α. We calculated the entanglement of formation of these states also and found that quantum discord δ is greater than or equal to the entanglement of formation E. The two are identical and equal to 0 at a = 0 and identical and equal to 1 at a = 1. D is zero in the beginning and then picks up.

For the other two states, which we call quasi-Werner states, however, the dependence of quantum discord D is seen on θ and α although it is independent of ϕ. For very small $|\alpha|$, the dependence on θ is very pronounced and D first increases with a and then it decreases. Dependence on θ, however, becomes unnoticeable for $|\alpha| > 1$. For this case, however, D increases uniformly with a. This is shown in the FIG. 12.

The behaviour for higher values of $|\alpha|^2$ is shown in FIG. 13.

Variation of minimum of quantum discord against θ and of the entanglement of formation E with $|\alpha|$ and a are shown in the FIG. 14.

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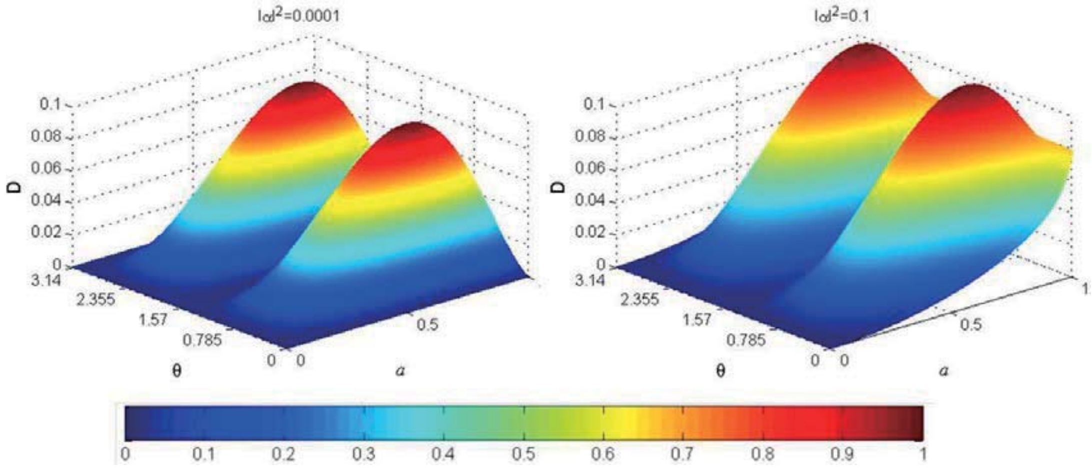

FIG. 12.

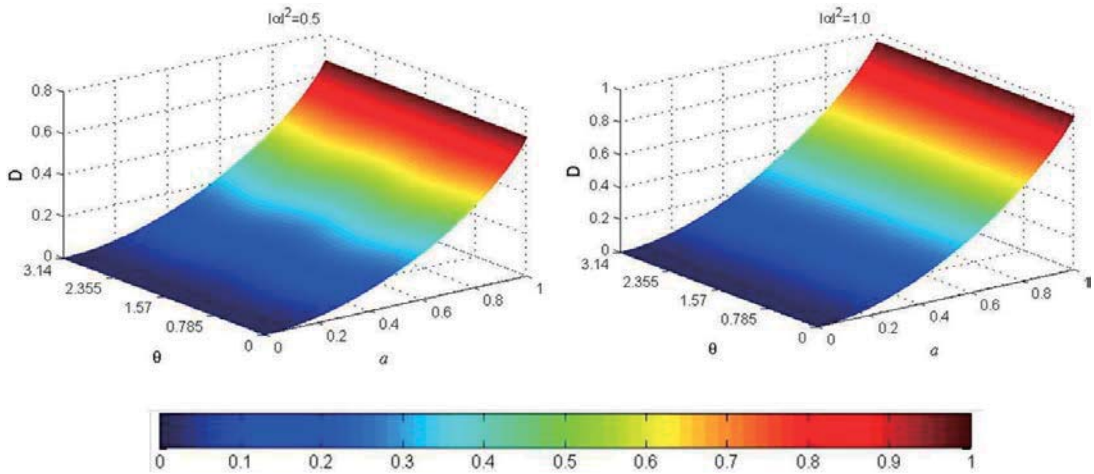

FIG. 13.

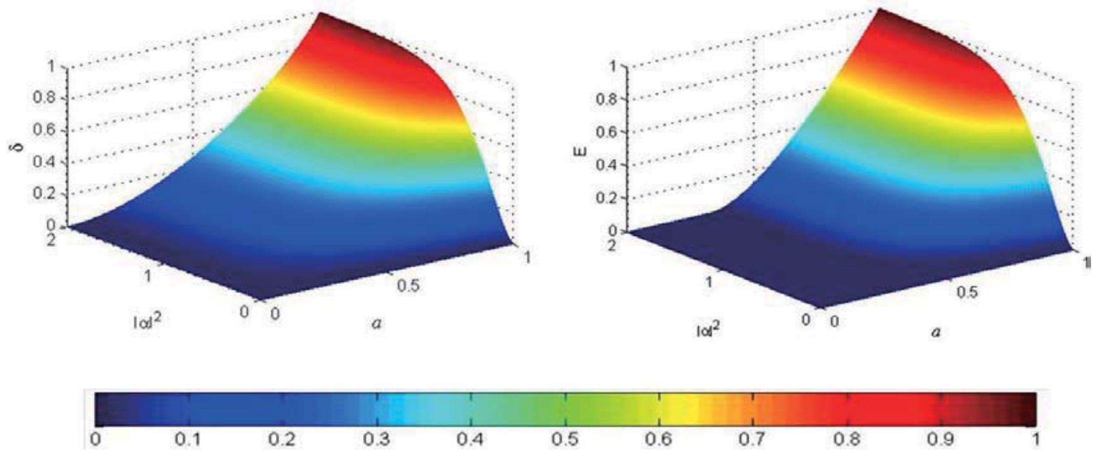

FIG. 14.