Forty-five years of entangled coherent states

Barry C. Sanders
Institute for Quantum Science and Technology, University of Calgary, Alberta T2N 1N4, Canada

Entangled coherent states date back to 1967, when they arose implicitly in a refutation of an axiomatic superselection principle added to quantum mechanics. Since its inception, the entangled coherent state, which is a superposition of multi-mode coherent states, has found applications in quantum reference frames, quantum information processing, quantum communication, and mathematical physics, and entangled coherent states have been generated experimentally. I provide an overview of this exciting area of research.

Coherent states were introduced by Schrödinger in 1926 and became indispensable to quantum optics [1–3], especially as a quantum state that can be regarded in many ways as being classical with quantum noise added. The superposition of (single-degree-of-freedom) coherent states, often called “Schrödinger cat states”, essentially referring to Schrödinger’s famous illustration of the absurdity of quantum mechanics seemingly predicting that even states of being alive or dead could be in a superposition [4], has been studied extensively [5]. These “cat states” have been produced experimentally but only for coherent states that are not far apart [6], and such states are often called “Schödinger kittens” [7].

Extending the “cat state” to a superposition of coherent states with more than one degree of freedom is straightforward, but the properties are much richer than for one degree of freedom due to the presence of entanglement [8]. The history of entangled coherent states appears in my recent review article, which I summarize here [9]. The first appearance of entangled coherent states was almost unnoticeable, appearing obscurely in a famous refutation of the seeming need for an extra quantum-mechanical axiom on superselection [10]. The entangled coherent state representation shows how to circumvent charge superposition [11].

Ironically entangled coherent states next appeared as a tool to understand superpositions of single-degree-of-freedom coherent states rather than being investigated in their own right [12]. The entangled coherent stated arises through introducing a second degree of freedom, or mode, to perform homodyne detection [13–15]. Instead of viewing the second mode as part of the measurement apparatus, the perspective that two modes are being measured reveals that entangled cohereont states are at the heart of this “cat state” detection apparatus.

The entangled coherent state continued to lurk obscurely in the literature, making itself felt indirectly through a distinct manifestation: the pair coherent state [16–18], which is a particular realization of the Barut-Girardello coherent state [19], has an entangled coherent state representation [17]. Eventually entangled coherent states were studied directly: how to create such states [20–23] and determining their characteristics [21]. The term “entangled coherent state” was coined in 1992 in a study of these states that focused on how to produce such states but moreover on their entanglement including a violation of a Bell type of inequality [24, 25]. This violation was for the few-photon limit; the many-photon limit Bell inequality violation was later introduced [26].

Various means for creating entangled coherent states were subsequently studied. The Kerr nonlinearity concept, prevalent in early entangled-coherent-state papers, combines the intensity-dependent self phase shift to make “cat states” with beam splitters to combine modes [24] or simply to have two-mode Kerr nonlinear evolution [20]. The ideal Kerr evolution preserves photon number so these entangled coherent states exhibit the same Poisson Fock number distribution as unentangled coherent states. In contrast entangled coherent states can be constructed that do not preserve the coherent state’s Poissonian number distribution. One well known example is the even entangled coherent states and its counterpart: the odd entangled coherent state [27, 28], which are two-degree-of-freedom extensions of single-degree-of-freedom even and odd coherent states [29]. Such states are not created from Kerr-type nonlinearities and require other means.

An early means to create an entangled coherent state without a Kerr nonlinearity was set in the context of cavity quantum electrodynamics. A single atom interacts sequentially with two independent cavities and prepares an entangled coherent state via measuring the state of the atom and building the state non-deterministically based on the measurement outcomes [30].

Entangled coherent states have been produced in the laboratory successfully via two-mode pulsed parametric amplification followed by photon subtraction [31]. The parametric amplifier produces a squeezed vacuum state, which approximates a small-amplitude “cat state”, known as a “kitten state”. The outputs are recombined at a beam splitter with part of the field being directed a post-selective photon counter, and this post-selective process prepares the entangled coherent state.

Although “cat states” and entangled coherent states are typically studied as equally weighted superpositions of coherent states, unequally weighted, or “unbalanced”, entangled coherent states can be approximately generated in a double-cavity system as well [32]. This scheme employs, however, a peculiar kind of nonlinear evolution that is reminiscent of the nonlinear evolution used to generate Titulaer-Glauber generalized coherent states [33–35].

These studies treated just two degrees of freedom. Studies of more than two degrees of freedom followed later [36–38]. These extra degrees of freedom enable analogues of orthogonal entangled states in the entangled-coherent-state context such as Greenberger-Horne-Zeilinger and W states [39] as well as cluster states [40–42]. Entangled coherent states can serve as resources in quantum computing and in quantum communication due to the inherent bipartite and multi-partite entanglement.

The two-degree-of-freedom even entangled coherent state are maximally entangled: they hold the same entanglement as a single maximally entangled pair of qubits [43]. Astonishingly a bipartite entangled coherent state produced the Kerr type of nonlinear evolution possesses arbitrarily large entanglement if the time of evolution is short but much less entanglement for long evolution times [44]. The entanglement resource can be teleported [45, 46] and used as a teleportation resource [43, 45, 47].

Coherent states are much more general than being Schrödinger-Glauber coherent states: many other types of coherent states exist. Coherent states are eigenstates of the harmonic-oscillator annihilation operator and can be generalized for other symmetries as eigenstates of other lowering ladder operators [19]. Another generalization corresponds to generalizing the Glauber-Sudarshan displacement operator to other group actions [48–50]. These generalized coherent states can be in superposition, thereby generalizing the notion of the “cat state”. Similarly superpositions of tensor products of generalized coherent states serve as generalized entangled coherent states [51]. Coherent states can be generalized through various actions such as the photon-added coherent state, which leads to the notion of entangled photon-added entangled coherent states [52].

Entangled coherent state applications have proven to be quite exciting. Not only do they serve as a resource for teleportation, as discussed above, but also for quantum networks [53, 54] and as a qubit for quantum information processing [55, 56]. In addition to quantum computing, quantum metrology is another promising avenue for entangled coherent state applications [57]. Entangled coherent states are known to outperform other popular two-mode entangled states in quantum metrology [58–61], Furthermore entangled coherent states are excellent for digital parameter discrimination [62].

ACKNOWLEDGMENTS

This project has been supported by AITF, CIFAR, NSERC, MITACS and PIMS.

[1] R. J. Glauber, Phys. Rev. Lett. **10**, 84 (1963).
[2] E. C. G. Sudarshan, Phys. Rev. Lett. **10**, 277 (1963).
[3] J. R. Klauder and B. S. Skagerstam, *Coherent States* (World Scientific, Singapore, 1985).
[4] E. Schrödinger, Naturwissenschaften **23**, 812 (1935).
[5] V. Bužek and P. L. Knight, Progress in Optics **34**, 1 (1995).
[6] M. Brune, E. Hagley, J. Dreyer, X. Maitre, A. Maali, C. Wunderlich, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **77**, 4887 (1996).
[7] A. Ourjoumtsev, R. Tualle-Brouri, J. Laurat, and P. Grangier, Science **312**, 83 (2006).
[8] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009).
[9] B. C. Sanders, J. Phys. A: Math. Theor. **45**, 244002 (2012).
[10] Y. Aharonov and L. Susskind, Phys. Rev. **155**, 1428 (1967).
[11] B. C. Sanders, S. D. Bartlett, T. Rudolph, and P. L. Knight, Phys. Rev. A **68**, 042329 (2003).
[12] B. Yurke and D. Stoler, Phys. Rev. Lett. **57**, 13 (1986).
[13] H. P. Yuen and J. H. Shapiro, IEEE Trans. Inf. Theory **26**, 78 (1980).
[14] H. P. Yuen and V. W. S. Chan, Opt. Lett. **8**, 177 (1983).
[15] T. Tyc and B. C. Sanders, J. Phys. A: Math. Gen. **37**, 7341 (2004).
[16] G. S. Agarwal, Phys. Rev. Lett. **57**, 827 (1986).
[17] G. S. Agarwal, J. Opt. Soc. Am. B **5**, 1940 (1988).
[18] G. S. Agarwal and K. Tara, Phys. Rev. A **43**, 492 (1991).
[19] A. O. Barut and L. Girardello, Commun. Math. Phys. **21**, 41 (1971).
[20] A. Mecozzi and P. Tombesi, Phys. Rev. Lett. **58**, 1055 (1987).
[21] P. Tombesi and A. Mecozzi, J. Opt. Soc. Am. B **4**, 1700 (1987).
[22] B. C. Sanders and D. A. Rice, Opt. Quant. Electronics **31**, 525 (1999).
[23] B. C. Sanders and D. A. Rice, Phys. Rev. A **61**, 013805 (2000).
[24] B. C. Sanders, Phys. Rev. A **45**, 6811 (1992).
[25] B. C. Sanders, Phys. Rev. A **46**, 2966 (1992).
[26] A. Mann, B. C. Sanders, and W. J. Munro, Phys. Rev. A **51**, 989 (1995).
[27] C.-l. Chai, Phys. Rev. A **46**, 7187 (1992).
[28] V. V. Dodonov, V. I. Man’ko, and D. E. Nikonov, Phys. Rev. A **51**, 3328 (1995).
[29] V. V. Dodonov, I. A. Malkin, and V. I. Man’ko, Physica **72**, 597 (1974).
[30] L. Davidovich, A. Maali, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. Lett. **71**, 2360 (1993).
[31] A. Ourjoumtsev, F. Ferreyrol, R. Tualle-Brouri, and P. Grangier, Nature Phys. **5**, 189 (2009).
[32] B. Wielinga and B. C. Sanders, J. Mod. Opt. **40**, 1923 (1993).
[33] U. M. Titulaer and R. J. Glauber, Phys. Rev. **145**, 1041 (1965).
[34] I. Bialynicki-Birula, Phys. Rev. **173**, 1207 (1968).
[35] D. Stoler, Phys. Rev. D **4**, 2309 (1971).

[36] I. Jex, P. Törmä, and S. Stenholm, J. Mod. Opt. **42**, 1377 (1995).
[37] S.-B. Zheng, Quant. Semiclass. Opt. B: J. European Opt. Soc. B **10**, 691 (1998).
[38] X. Wang and X. Sanders, Phys. Rev. A **65**, 012303 (2001).
[39] H. Jeong and N. B. An, Phys. Rev. A **74**, 022104 (2006).
[40] P. P. Munhoz, F. L. Semião, and Vidiello, Phys. Lett. A **372**, 3580 (2008).
[41] E. M. Becerra-Castro, W. B. Cardoso, A. T. Avelar, and B. Baseia, J. Phys. B: At. Mol. Opt. Phys. **41**, 085505 (2008).
[42] W.-F. Wang, X.-Y. Sun, and X.-B. Luo, Chin. Phys. Lett. **25**, 839 (2008).
[43] S. J. van Enk and O. Hirota, Phys. Rev. A **64**, 022313 (2001).
[44] S. J. van Enk, Phys. Rev. Lett. **91**, 017902 (2003).
[45] X. Wang, Phys. Rev. A **64**, 022302 (2001).
[46] T. J. Johnson, S. D. Bartlett, and B. C. Sanders, Phys. Rev. A **66**, 042326 (2002).
[47] H. Jeong, M. S. Kim, and J. Lee, Phys. Rev. A **64**, 052308 (2001).
[48] F. T. Arecchi, E. Courtens, R. Gilmore, and H. Thomas, Phys. Rev. A **6**, 2211 (1972).
[49] R. Gilmore, Ann. Phys. (N.Y.) **74**, 391 (1972).
[50] A. M. Perelomov, Commun. Math. Phys. **26**, 222 (1972).
[51] X. Wang, B. C. Sanders, and S.-h. Pan, J. Phys. A: Math. Gen. **33**, 7451 (2000).
[52] M. X. Tao, H. Lu, and W. L. She, Acta Physica Sinica **51**, 1996 (2002).
[53] P. van Loock, N. Lütkenhaus, W. J. Munro, and K. Nemoto, Phys. Rev. A **78**, 062319 (2008).
[54] A. El Allati, Y. Hassouni, and N. Metwally, Phys. Scr. **83**, 065002 (2011).
[55] P. T. Cochrane, G. J. Milburn, and W. J. Munro, Phys. Rev. A **59**, 2631 (1999).
[56] M. C. de Oliveira and W. J. Munro, Phys. Rev. A **61**, 042309 (2000).
[57] N. A. Ansari, L. D. Fiore, M. A. Man'ko, V. I. Man'ko, S. Solimeno, and F. Zaccaria, Phys. Rev. A **49**, 2151 (1994).
[58] C. C. Gerry and R. A. Campos, Phys. Rev. A **64**, 063814 (2001).
[59] C. C. Gerry, A. Benmoussa, and R. A. Campos, Phys. Rev. A **66**, 013804 (2002).
[60] J. Joo, W. J. Munro, and T. P. Spiller, Phys. Rev. Lett. **107**, 083601 (2011).
[61] J. Joo, W. J. Munro, and T. P. Spiller, Phys. Rev. Lett. **107**, 219902 (2011).
[62] O. Hirota, K. Kato, and D. Murakami, *Effectiveness of entangled coherent state in quantum metrology*, arXiv:1108.1517 (2011).