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Tamagawa University Quantum ICT Research Institute Bulletin, Vol.5, No.1, 5-10, 2015

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# Towards Quantum Enigma Cipher -A protocol for G bit/sec encryption based on discrimination property of non-orthogonal quantum states-

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**Abstract**—This research note suggests a new way to realize a high speed direct encryption based on quantum detection theory. The conventional cipher is designed by a mathematical algorithm and its security is evaluated by the complexity of the algorithm for cryptanalysis and ability of computers. This kind of cipher cannot exceed the Shannon limit of cryptography, and it can be decrypted with probability one in principle by trying all the possible keys against the data length equal to the secret key length. A cipher with quantum effect in physical layer may exceed the Shannon limit of cryptography. The quantum stream cipher by  $\alpha/\eta$  or Yuen-2000 protocol (Y-00) which operates at Gbit/sec is a typical example of such a cipher. That is, ciphertext of mathematical cipher with a secret key is masked by quantum noise of laser light when an eavesdropper observes optical signals as a ciphertext of the mathematical cipher, while the legitimate receiver does not suffer the quantum noise effect. As a result, the inherent difference of accuracy of ciphertext between eavesdropper and legitimate receiver arises. This is a necessary condition to exceed the Shannon limit of cryptography. In this note, we present a new method to generate an inherent difference of accuracy of the ciphertext, taking into account a fundamental properties of quantum detection schemes.

## I. INTRODUCTION

I introduced, in several public talks, the following situation: [A new network scheme so called “Cloud computing system” based on data centers has recently attracted considerable attention. In that system, all data are communicated via a high speed optical network between a customer and data center or between data centers. There is a serious threat so called “Eavesdropper data center business”, which means the eavesdropper can get all data from the transmission line and sell specific data selected by the protocol analyzer to malicious people who want to get the secret data. This is a new business model of hacker in the era of cloud computing system.] Surprisingly, it was exposed in 2013 by E.J.Snowden that US and UK governments had collected all data from optical network.

So one needs to consider “Cyber attack against Layer-1 (physical layer)”. Technology of coupler for tapping has been developed by several institutes [1]. In addition,

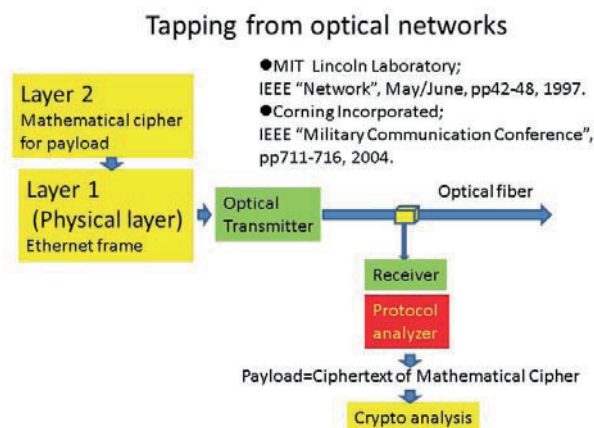


Fig. 1. Cyber attack against physical layer. Technology of coupler for tapping has been developed by several institutes. In addition, there are many optical monitor ports for network maintenance. Thus, physical layer of high speed data link is a defenseless.

there are many optical monitor ports for network maintenance. Indeed physical layer of high speed data link is a defenseless. So, one can obtain the correct ciphertext of mathematical cipher for payload at Layer-2, and one can store it at memory.

This fact drove us to develop a special physical cipher for optical domain which protects ultra high speed data at Layer-1(physical layer). The requirements on the encryption system are as follows:

### Requirement of specifications:

- (1) Data-speed:1 Gbit/sec ~ 100 Gbit/sec
- (2) Distance: 1000 Km ~ 10000 Km
- (3) Encryption scheme: Symmetric Key Cipher
- (4) Security: Provable security, Secure against Brute force attack( exhaustive search trial for secret key) by means

of computer and also physical devices.

So far, the standard encryption systems based on purely mathematical algorithm have been employed to ensure security of data and secret key. However, it is still difficult to quantitatively guarantee the security. Furthermore the eavesdropper can store the correct ciphertext from communication lines as denoted at above. So one cannot deny the possibility of the decipherment of stored ciphertext by the discovery of the mathematical algorithm, or by the development of high speed computers.

The emerging development of physical cipher at the physical layer suggests a new way of building secure cloud computing system. A new concept of random cipher based on quantum noise has been proposed [2]. It is called quantum noise randomized stream cipher or simply quantum stream cipher. The representative concrete protocol is  $\alpha/\eta$  or Y-00, and several implementation schemes have been realized [3,4,5]. The most important feature of this physical cipher is that the eavesdropper cannot get the correct ciphertext of mathematical cipher, for example a stream cipher by PRNG (pseudo random number generator), from communication lines, while the legitimate user can get it based on a knowledge of secret key of PRNG. Thus, the ciphertext:  $Y^B$  of the legitimate user and the ciphertext:  $Y^E$  of the eavesdropper may be different as  $Y^B \neq Y^E$ . This can be enhanced by additional randomization such as Hirota-Kurosawa method [6]. Furthermore, the ciphertext of the eavesdropper becomes random by the real noise in her receiver. The security is guaranteed by performance limitation of the eavesdropper's receiver to get ciphertext and cryptanalysis ability to ciphertext. A role of the Quantum Mechanics is to provide restrictions to accuracy of eavesdropper's ciphertext in physical layer, and to prevent, by physical law such as no-cloning theorem, eavesdropper's attempt of the brute force attack. In addition, there is a physical size limitation of devices for the physical brute force attack, which is also law of physics.

This type of scheme opens a new paradigm in the cryptology for physical layer. Indeed one can realize a cipher exceeding the Shannon limit of cryptography so called "Quantum Enigma Cipher" that has been defined in our article[7]. However, one needs to investigate several randomization methods based on quantum mechanics.

In the following sections, we will discuss a new method to realize a randomization of ciphertext based on quantum detection theory.

## II. INFORMATION THEORETIC BASIS FOR SECURITY OF PHYSICAL STREAM CIPHER

It is well known that the Shannon limit of the symmetric key cipher derives a pessimistic theorem which justifies the one time pad. If the Shannon limit can be exceeded, one does not need hybrid cipher of QKD and one time pad. In this section, we give a survey of a new

type of information theoretic basis on the symmetric key cipher.

In the conventional cipher, the ciphertext  $Y$  is determined by the information bit  $X$  and running key  $K_R$  extended by PRNG with short initial key. This is called non random cipher. On the other hand, one can introduce more general cipher system so called random cipher by classical noise such that the ciphertext is defined as follows:

$$Y_n = f(X_n, K_{R(n)}, r_n) \quad (1)$$

where  $r_n$  is noise. However, in the Shannon theory for non random or random cipher, the information theoretic security on data is given as follows:

**Theorem 1:**(Shannon, 1949 [8])

The information theoretic security against ciphertext only attack on data has the following limit.

$$H(X|Y) \leq H(K_s) \quad (2)$$

This is called Shannon limit for the symmetric key cipher. In addition, the information theoretic security on key is given by

$$H(K_s|Y) \leq H(K_s) \quad (3)$$

Although the equality of Eq(2),(3) can be realized by ideal one time pad, there is no way to exceed the Shannon limit in the conventional encryption schemes. To exceed the Shannon limit is also essential for secure fresh key generation by communication or information theoretic security against known plaintext attack in the symmetric key cipher.

A random cipher by noise may provide a new category of the cipher, and so far many randomized stream ciphers have been proposed in the literature of cryptology. However, at present, there does not exist an attractive cipher. The reason comes from the fact that the ciphertext of eavesdropper and that of legitimate user are the same in the classical system.

It has been pointed out that there exists an attractive cipher, which gets out the frame of the Shannon theory of cryptology, by means of the combination of the concept of the private randomization of C.F.Gauss and the optical-quantum communication system [2]. The crucial property of such a cipher is that the ciphertexts of eavesdropper and legitimate user may be different. Then it has a potential to exceed the Shannon limit, and may lead to realization of "Quantum Enigma Cipher"[7]. In the following, we will give short review on the basis of cipher that exceeds the Shannon limit.

**Theorem 2:** Let us denote the received ciphertexts of legitimate receiver and eavesdropper as follows:

$$\begin{aligned} Y_n^B &= \{y_1^B, y_2^B, y_3^B, \dots\}, \\ Y_n^E &= \{y_1^E, y_2^E, y_3^E, \dots\} \end{aligned} \quad (4)$$

The necessary condition to exceed the Shannon limit is that the ciphertext of eavesdropper and that of legitimate user are different [for example, see 9].

$$Y_n^B = f(X_n, K_{R(n)}, r_n^B) \neq Y_n^E = f(X_n, K_{R(n)}, r_n^E) \quad (5)$$

and

$$P_e(Y_n^E) \gg P_e(Y_n^B) \quad (6)$$

which is a condition of the performance on an error of detection of ciphertext for eavesdropper and legitimate receivers.

Still the sufficient condition to exceed the Shannon limit is not strict, but if the following relation is retained, one can say that the cipher exceeds the Shannon limit.

$$H(X|Y^E, K_s) > H(X|Y^B, K_s) = 0 \quad (7)$$

$(Y^E, K_s)$  means that key is given after measurement. The above equation means that eavesdropper cannot pin down the information bit even if she gets a secret key or unknown parameter for eavesdropper after her measurement of the ciphertext as physical signals.

### III. A NEW QUANTUM STREAM CIPHER PROTOCOL BASED ON CONTROLLED A PRIORI PROBABILITY

According to the previous section, an important concept to exceed the Shannon limit is to realize the difference of accuracy of ciphertext between an eavesdropper's receiver and a legitimate receiver. In this section, we will propose a new way to realize it.

#### A. Protocol

A typical encryption scheme for physical layer in the network is a physical cipher that scrambles transmission signals by integrating mathematical cipher with physical phenomena. So we employ a mathematical encryption box and a physical encryption box. This concept has a similarity to Triple DES with three different keys in classical cipher. Let us show our method as follows:

(1) Here the data sequence is encrypted by PRNG with short secret key:  $K_s$ . This part is an input to the physical encryption box:  $Enc(\mathcal{P})$ .

(2) The ciphertext generated from PRNG is translated into  $M$ -ary signals.

(3) An additional scheme as physical cipher is only to control a priori probability of  $M$ -ary signals by certain control box:  $Enc(\mathcal{P})$ . The signals from  $Enc(\mathcal{P})$  are mapped on " $M$ -ary asymmetry non-orthogonal quantum states" at laser light transmitter, corresponding to one to one.  $\mathcal{P}$  is a physical secret key of physical cipher. Thus, the legitimate receiver knows a priori probability, but the eavesdropper does not.

(4) The eavesdropper's receiver and the legitimate receiver will employ quantum optimum  $M$ -ary detection receiver. Since the legitimate receiver knows a priori probability, and the eavesdropper does not, the legitimate receiver can employ quantum Bayes strategy, but the eavesdropper has to employ quantum minimax strategy.

In general, the performance of quantum Bayes strategy is better than that of quantum minimax strategy. This means that error of the legitimate receiver is smaller than that of the eavesdropper. That is, we have

#### Theorem 3:

$$P_e(\text{Bayes}) \leq P_e(\text{minimax}) \quad (8)$$

By using this difference, the legitimate users can establish secure direct communication, because the accuracy of the eavesdropper's ciphertext is deteriorated. Let us give such a principle in the following.

#### B. Quantum detection theory

Let us first describe the theory of quantum Bayes criterion. The optimum condition is given as follows:

The evaluation function of quantum Beys strategy is as follows:

$$\min_{\Pi} \sum_i \sum_j \xi_i C_{ji} Tr \rho_i \Pi_i \quad (9)$$

where,  $\Pi = \{\Pi_j\}$  is POVM(positive operator valued measure). As usual, we define the risk operator as follows:

$$W_j \equiv \sum_{i=1}^M \xi_i C_{ji} \rho_i \quad (10)$$

$$\Gamma = \sum_{j=1}^M \Pi_j W_j = \sum_{j=1}^M W_j \Pi_j \quad (11)$$

Here, we assume that  $C_{ji} = 1$  ( $i \neq j$ ),  $C_{ji} = 0$  ( $i = j$ ). Then the criterion becomes average error probability.

$$\min_{\Pi} P_e = \min_{\Pi} (1 - \sum_i \xi_i Tr \rho_i \Pi_i) \quad (12)$$

The optimum condition for quantum Bayes strategy with respect to POVM had been formulated by Holevo[10] and Yuen[11], respectively.

**Theorem 4:** Optimum conditions for POVM of quantum Bayes strategy are given as follows:

$$\begin{aligned} (W_j - \Gamma)\Pi_j &= \Pi_j(W_j - \Gamma) = 0, \quad \forall j \\ \Pi_j(W_i - W_j)\Pi_i &= 0, \quad \forall i, j \\ W_j - \Gamma &\geq 0, \quad \forall j \end{aligned} \quad (13)$$

where

$$W_j \equiv \sum_{i=1}^M \xi_i C_{ji} \rho_i \quad (14)$$

$$\Gamma = \sum_{j=1}^M \Pi_j W_j = \sum_{j=1}^M W_j \Pi_j \quad (15)$$

In general, when a priori probability is unknown, one cannot apply Bayes strategy, and minimax strategy may be employed. In the following, we introduce a quantum strategy so called quantum minimax strategy which was formulated by Hirota-Ikehara in 1978.

In quantum case, the criterion of quantum minimax strategy is given by

$$\bar{C}_m = \min_{\{\Pi_j\}} \cdot \max_{\{\xi_i\}} \sum_{i=1}^M \sum_{j=1}^M \xi_i C_{ji} \text{Tr} \rho_i \Pi_j \quad (16)$$

where  $C_{ji} = 1$  ( $i \neq j$ ),  $C_{ji} = 0$  ( $i = j$ ). Then the criterion becomes

$$P_{em} = \min_{\{\Pi_j\}} \cdot \max_{\{\xi_i\}} \left\{ 1 - \sum_{i=1}^M \xi_i \text{Tr} \rho_i \Pi_j \right\} \quad (17)$$

A primitive analysis was published in 1982 [12] as follows:

**Theorem 5:** Let  $\{\xi_i\}$  and  $\{\Pi_j\}$  be a priori probability and POVM, respectively. Then we have

$$\min_{\{\Pi_j\}} \cdot \max_{\{\xi_i\}} P_e = \max_{\{\xi_i\}} \cdot \min_{\{\Pi_j\}} P_e \quad (18)$$

**Theorem 6:** The optimum conditions for POVM is given by

$$\begin{aligned} \text{Tr} \rho_i \Pi_i &= \text{Tr} \rho_j \Pi_j, & \forall i, j \\ (W_j - \Gamma) \Pi_j &= \Pi_j (W_j - \Gamma) = 0, & \forall j \\ \Pi_j (W_i - W_j) \Pi_i &= 0, & \forall i, j \\ W_j - \Gamma &\geq 0, & \forall j \end{aligned} \quad (19)$$

Recently, the mathematical progress for quantum minimax theory has been given by G.M.D'Ariano et al[13], K.Kato [14], F.Tanaka [15], and K.Nakahira et al [16].

Now we can easily show the proof of the theorem 3. The space  $\Omega$  of a priori probability is separable and the space  $\mathcal{D}$  of POVM is compact. Hence this strategy problem as the zero sum two person game becomes strictly determined from Wald's theorem. So, the minimax value given by such two person game is the quantum Bayes value with the worst a priori probability. The minimax value is always larger than the quantum Bayes value.

Thus, a knowledge of a priori probability can be employed as an encryption key in physical layer.

#### IV. CONCRETE PROPERTIES OF DIFFERENCE BETWEEN BAYES AND MINIMAX

##### A. Design of quantum state signals

Let  $\{\rho_j\}$  be an ensemble of quantum states and whole element states are non-orthogonal each other of a set of states. In addition, let us assume that a geometrical structure is an asymmetric.

**Definition 1:** Symmetric quantum state is defined by

$$\rho_j = (U^\dagger)^{j-1} \rho_1 (U)^{j-1}, \quad j = 1, 2, 3, \dots, M \quad (20)$$

where  $U$  is a certain unitary operator. The example is as follows: Let  $|\alpha\rangle$  be coherent state. The symmetric three states are

$$|\alpha \exp(i\pi/2)\rangle, |\alpha \exp(i7\pi/6)\rangle, |\alpha \exp(i11\pi/6)\rangle \quad (21)$$

like three pointed star of Mercedes-Benz.

**Definition 2:** Asymmetric quantum state is defined by

$$\rho_j \neq (U^\dagger)^{j-1} \rho_1 (U)^{j-1}, \quad j = 1, 2, 3, \dots, M \quad (22)$$

An example of the asymmetric state is

$$|\alpha \exp(i\pi/2)\rangle, |\alpha \exp(i7\pi/6)\rangle, |\alpha \exp(i(\theta + 11\pi/6))\rangle \quad (23)$$

where  $\theta \neq 0$ . Here we have open problem on the optimum condition to design an efficient model for real use as follows:

$$\eta = \max_{\mathcal{R}} \max_{\mathcal{P}} \frac{P_e(\text{minimax})}{P_e(\text{Bayes})} \quad (24)$$

where  $\sigma^{(k)} = \{\rho_j^{(k)}\} : j = 1, 2, 3, \dots, M$ ,  $\mathcal{R} = \{\sigma^{(k)}\} : k = 1, 2, 3, \dots$ ,  $\Omega^{(k)} = \{\xi_j^{(k)}\} : j = 1, 2, 3, \dots, M$ , and  $\mathcal{P} = \{\Omega^{(k)}\} : k = 1, 2, 3, \dots$ .

Although we do not have an answer on the above question, we can show some examples.

##### B. Nakahira-Kato-Usuda iterative method for optimum solution

In general, the calculation of optimization problem in quantum detection theory is very hard, when a system consisting of a priori probability and a set of quantum states is given. So the iterative methods have been proposed by many researchers [17,18]. Recently, Nakahira-Kato-Usuda have provided useful iterative method which is applicable to both quantum Bayes and quantum minimax strategy [19]. As an example, Nakahira demonstrated properties for coherent state PSK(phase shift keying) of  $M=3$  [20]. He showed

$$\frac{P_e(\text{minimax})}{P_e(\text{Bayes})} \sim 10 \quad (25)$$

where  $\langle n \rangle = |\alpha|^2 = 1$ . Although one needs to clarify the case of  $\langle n \rangle = 10^4 \sim 10^6$  at transmitter and  $M \gg 1$ , the above example is useful for estimation of the performance of this protocol. More realistic case will be reported in the future articles.



### C. Conjecture of good performance

The eavesdropper can make a tapping at close the transmitter, so one needs to take into account the transmission loss for the performance advantage of the legitimate receiver. The quadrature amplitude modulation (QAM) is the most efficient modulation scheme under the average power constraint [21]. Keeping the good error performance of the legitimate receiver, one can design the QAM signal constellation such that the error performance of the eavesdropper's receiver becomes the worst. We will report the concrete design in the next paper.

### V. SECURITY OF TOTAL SYSTEM

Quantum enigma cipher which is the generalized symmetric key cipher as a physical cipher is designed by utilizing the features of mathematical encryption and physical phenomena in transmission line. The task of the eavesdropper is to find a secret key of mathematical encryption box from ciphertext as physical signal. However, the eavesdropper cannot get the correct ciphertext. So it prevents the mathematical analysis to search the secret key and the Brute force attack (exhaustive search trial for secret key) by computer. Although the eavesdropper may employ a physical Brute force attack, she cannot realize it because of the law of physics such as several quantum no-go theorems or physical size limitation for devices to try  $2^{|K_S|}$  (candidate of the correct key), where  $|K_S|$  is key length of secret key of mathematical encryption box. Thus, one can realize a cipher for physical layer such that the security of the total system does not depend on the ability of computers and mathematical algorithm.

Here let us estimate a performance of the quantitative security. It is usual in the cryptology to use guessing probability when the security does not depend on computational ability.

(1) QKD(with initial secret key)+One Time Pad:

Let the generated key length  $|K_G|$  by QKD and the data length of one time pad be both  $10^4$  bits. The guessing probability of the key sequence of  $10^4$  for one time pad in the real case is given by

$$\begin{aligned} P(K_G|Y) &\leq \frac{1}{2^{|K_G|}} + d \\ &\sim 10^{-15} \end{aligned} \quad (26)$$

where  $d$  is the trace distance in QKD model. The data transmission speed is limited by the speed of QKD.

(2) Quantum Enigma Cipher:

Let the secret key length  $|K_s|$  of mathematical encryption box and the data length be 256 bits and  $10^4 \ll |X| \leq 10^{|K_s|} \sim 10^{70}$  bits, respectively. The guessing probability of the secret key in the optimum design may be expected as follows:

$$P(K_s|Y) \sim 10^{-70} \quad (27)$$

The data transmission speed is limited by the speed of mathematical encryption and the speed of the conventional optical communication system. Thus, the quantum

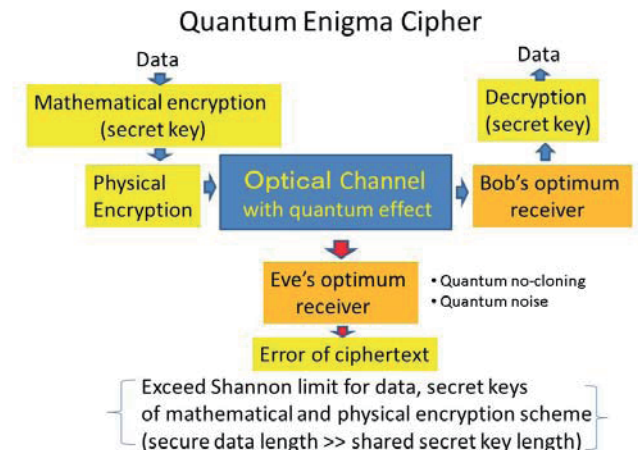


Fig. 2. Basic concept of quantum enigma cipher

enigma cipher has a potential for providing the ultimate encryption scheme in high speed optical data link.

### VI. CONCLUSION

Y-00 type quantum stream cipher as a typical example of physical cipher in physical layer already has been realized and has been applied to 1 Gbit/sec ~ 100 Gbit/sec real optical transmission system [22][23]. Now, we are concerned with a scheme with the ultimate security such as Quantum Enigma Cipher. There are two ways. One is to generalize Y-00 type stream cipher by additional physical randomizations, and other is to employ a new protocol. In this note, to stimulate a development of quantum enigma cipher, we have introduced a new protocol that is useful for constructing it. More detailed story including a compound system of Y-00 type and this method will be introduced in the conference on Quantum Physics and Nuclear Engineering held in London at March 2016.

### ACKNOWLEDGMENT

I am grateful to K.Nakahira, K.Kato, T.Usuda, and F.Futami for fruitful discussions, to Mercedes-Benz club and Ministry of Defense.

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