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***Abstract*—Two-mode squeezed light is an important quantum entanglement resource for realizing quantum radar based on the quantum illumination method. It can be generated by mixing two single-mode squeezed light beams by a beam splitter with controlling a relative optical phase at 90 degrees. From an engineering perspective of quantum radar, it is important to evaluate how much minute control of the optical phase is required at around 90 degrees to maintain the quantum entanglement of the light source. In this article correlation variances of quadrature phase amplitudes of two-mode squeezed light beams are derived as a function of the optical phase. Then the controllability of the optical phase to maintain the quantum entanglement is analyzed based on Duan's and Simon's inseparability criterion, especially in the case of high-power two-mode squeezed light beams. An effect of optical losses on the quantum entanglement and the optical-phase controllability of the light source is also discussed.**

I. Introduction

Quantum illumination is a recently developed target detection method utilizing quantum entanglement and expected to be applied to quantum radar [1], [2], [3]. Initially a pair of entangled photons was proposed as a quantum entanglement resource [1]. One of the entangled photons is transmitted towards a target and reflected to a radar receiver. The other photon is directly sent to the receiver with a safe channel. The quantum correlation between entangled photons is exploited to improve error probability of discrimination for target presence or absence even in a lossy and noisy environment. Tan, *et al.* developed quantum illumination method with entangled two-mode Gaussian states such as two-mode squeezed light and showed improvement in error probability of target detection compared with a single-mode coherent state [2]. So, the author is interested in applying a two-mode squeezed light source to quantum radar based on the quantum illumination method [4].

Two-mode squeezed light is a macroscopic quantum entangled state of electro-magnetic fields and shows non-classical correlation between quadrature phase amplitudes in each optical beam. It is often generated by mixing two independent single-mode squeezed light beams using a beam splitter with transmissivity of 0.5. In previous experimental works, a relative optical phase between single-mode squeezed light beams was usually controlled at 90 degrees to generate quantum entanglement [5]. From an engineering perspective of quantum radar, it is important to evaluate how much minute control of the optical phase is needed at around 90 degrees of angular range, especially in the case of high-power two-mode squeezed light. Furthermore, it is also important to evaluate how much the controllability of the optical phase and quantum entanglement of the light source are affected by external environments. In the quantum radar system one of the entangled beams is transmitted through a turbulent atmosphere or a lossy optical medium. These environmental factors degrade quantum entanglement of the light source and affects performance of target detection.

In this article the author studied the controllability of optical phase of high-power two-mode squeezed light beams in a lossy optical medium which is one of the most probable examples of environmental factors. To verify quantum entanglement of the light source, an inseparability criterion is necessary. One useful and practical inseparability criterion for continuous-variable entangled states was developed by Duan, *et al.* [6] and Simon [7]. This criterion is based on a correlation variance of quadrature phase amplitudes of the light source which can be directly detected by balanced homodyne measurement. Therefore it has been frequently used in continuous-variable quantum-optics experiments to verify quantum entanglement [5]. In this article correlation variances of quadrature phase amplitudes of two-mode squeezed light beams are derived as a function of relative optical phase and optical losses in environments. Then the optical-phase controllability is discussed especially in the case of the high-power two-mode squeezed light source based on the inseparability criterion. The effects of optical losses on the optical-phase controllability of the light source are also discussed.

II. Optical model

In this section a description of two-mode squeezed light is derived using Heisenberg representation (with unit of $\hbar = 1/2$, i.e. $[\hat{x}, \hat{y}] = i/2$) as is often used in continuous-variable quantum-optics experiments [5]. As shown in Fig. 1 entangled two-mode squeezed light beams are often generated by combining two single-mode squeezed light beams SQ_1 and SQ_2 with a beam splitter BS(0.5) with transmissivity of 0.5. ϕ is a relative optical phase between squeezed light beams SQ_1 and SQ_2.

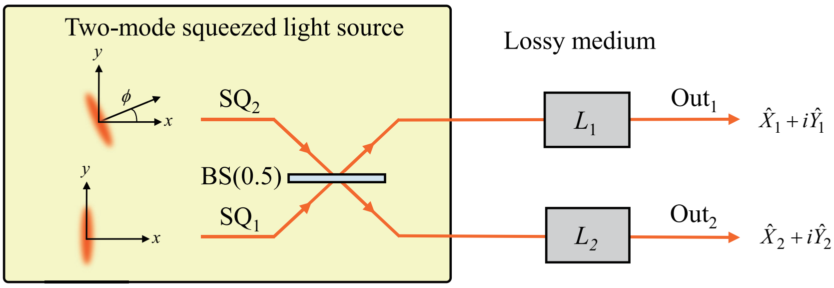

Fig. 1. Optical model for generating two-mode squeezed light beams. Incident single-mode squeezed light beams SQ$_1$ and SQ$_2$are combined using a beam splitter BS(0.5) with transmissivity of 0.5 with relative optical phase ϕ between them. Two entangled outputs Out$_1$ and Out$_2$ are affected by an environment with optical losses L_1 and L_2 in each optical path.

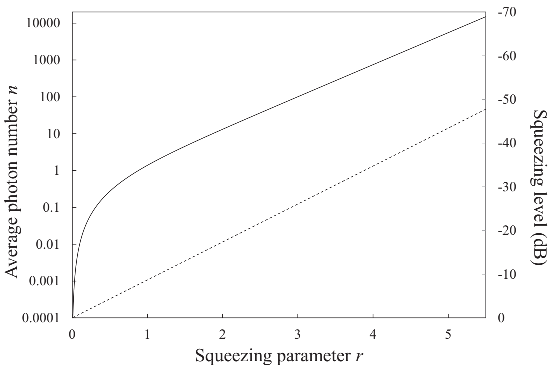

Fig. 2. Relation between squeezing parameter r and average photon number n (shown by solid line), and squeezing level (shown by dotted line).

We write complex amplitude operators of input squeezed light beams SQ$_1$ and SQ$_2$ as

$$\hat{a}_1 = e^{-r}\hat{x}_1 + ie^{r}\hat{y}_1 \tag{1}$$

and

$$\hat{a}_2 = e^{i\phi}(e^{-r}\hat{x}_2 + ie^{r}\hat{y}_2). \tag{2}$$

$\hat{x}_i$ and $\hat{y}_i$ $(i = 1, 2)$ are quadrature phase amplitude operators of incident vacuum states for two independent squeezers which are not shown in Fig. 1. Incident single-mode squeezed light beams SQ$_1$ and SQ$_2$ are characterized by squeezing parameter r which is an important experimental parameter and corresponds to optical gain of squeezers. Squeezing parameter r is related with average photon number n of incident squeezed light beams SQ$_1$ and SQ$_2$ by an equation of $n = \sinh^2 r$. Squeezing parameter r is also related with squeezing level (dB) of incident squeezed light beams which experimentalists usually use and expressed as $10\log_{10}e^{-2r}$. Fig. 2 shows calculation results of average photon number n by a solid line and squeezing level by a dotted line as a function of squeezing parameter r. In current experimental works of squeezed light generation, -12.7 dB of squeezing level is reported [8]. It corresponds to squeezing parameter r of 1.46 and average photon number n of 4.2.

In the quantum radar entangled beams are transmitted in a lossy optical medium. We introduce optical losses L_1 and L_2 in both optical paths of entangled output beams as shown in Fig. 1. These optical losses are modeled as mixture of vacuum states through beam splitters with amplitude transmissivity of $\sqrt{1-L_1}$ and $\sqrt{1-L_2}$, respectively. To derive input-output relation we use quadrature phase amplitude operators of vacuum states Vac_1 ($\hat{x}'_1$, $\hat{y}'_1$) and Vac_2 ($\hat{x}'_2$, $\hat{y}'_2$) which contaminate to the output states, respectively. Finally complex amplitude operators of entangled outputs Out$_1$ and Out$_2$ are given as

$$\begin{aligned}\hat{A}_1 &= \hat{X}_1 + i\hat{Y}_1 \\ &\frac{\sqrt{1-L_1}}{\sqrt{2}}\left(e^{-r}\hat{x}_1 + \cos\phi e^{-r}\hat{x}_2 - \sin\phi e^{r}\hat{y}_2\right) + \sqrt{L_1}\hat{x}'_1 \\ &+ i\left\{\frac{\sqrt{1-L_1}}{\sqrt{2}}\left(e^{r}\hat{y}_1 + \sin\phi e^{-r}\hat{x}_2 + \cos\phi e^{r}\hat{y}_2\right) + \sqrt{L_1}\hat{y}'_1\right\}\end{aligned} \tag{3}$$

and

$$\begin{aligned}\hat{A}_2 &= \hat{X}_2 + i\hat{Y}_2 \\ &\frac{\sqrt{1-L_2}}{\sqrt{2}}\left(-e^{-r}\hat{x}_1 + \cos\phi e^{-r}\hat{x}_2 - \sin\phi e^{r}\hat{y}_2\right) + \sqrt{L_2}\hat{x}'_2 \\ &+ i\left\{\frac{\sqrt{1-L_2}}{\sqrt{2}}\left(-e^{r}\hat{y}_1 + \sin\phi e^{-r}\hat{x}_2 + \cos\phi e^{r}\hat{y}_2\right) + \sqrt{L_2}\hat{y}'_2\right\}.\end{aligned} \tag{4}$$

Correlation variance of quadrature phase amplitudes is defined as

$$\Delta_{1,2}{}^2 = \langle[\Delta(\hat{X}_1 - \hat{X}_2)]^2\rangle + \langle[\Delta(\hat{Y}_1 + \hat{Y}_2)]^2\rangle. \tag{5}$$

It has been proven by Duan, *et al.* [6] and Simon [7] that two outputs Out$_1$ and Out$_2$ are inseparable and show quantum entanglement when the relation

$$\Delta_{1,2}{}^2 < 1 \tag{6}$$

is satisfied. Equation (6) is a sufficient condition of quantum entanglement and therefore called as an inseparability criterion. It seems that there is a relation between correlation variance $\Delta_{1,2}{}^2$ and quantum entanglement

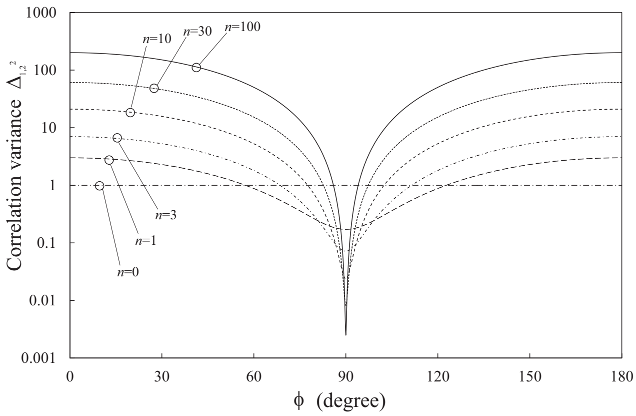

Fig. 3. Calculation results of correlation variance $\Delta_{1,2}{}^2$ with average photon number n of 100, 30, 10, 3, 1, and 0, respectively. In these calculations both optical losses L_1 and L_2 are assumed zero.

measure. However the quantative relation between correlation variance $\Delta_{1,2}{}^2$ and the strength of quantum entanglement is not clear at this moment.

In our optical model, correlation variance $\Delta_{1,2}{}^2$ can be calculated using quadrature phase amplitudes of final output states Out_1 and Out_2 as

$$\begin{aligned}\Delta_{1,2}{}^2 = \frac{1}{4}\Big\{&\frac{1}{2}(\sqrt{1-L_1}+\sqrt{1-L_2})^2(1+\sin^2\phi)e^{-2r}\\&+\frac{1}{2}(\sqrt{1-L_1}-\sqrt{1-L_2})^2\cos^2\phi e^{-2r}\\&+\frac{1}{2}(\sqrt{1-L_1}-\sqrt{1-L_2})^2(1+\sin^2\phi)e^{2r}\\&+\frac{1}{2}(\sqrt{1-L_1}+\sqrt{1-L_2})^2\cos^2\phi e^{2r}\Big\}\\&+\frac{1}{2}(L_1+L_2)\end{aligned} \tag{7}$$

and are given as a function of optical phase ϕ, squeezing parameter r and optical losses L_1 and L_2.

III. Calculation results

Fig. 3 shows calculation results of correlation variance $\Delta_{1,2}{}^2$ with average photon number n of 100, 30, 10, 3, 1, and 0 in the ideal case that both optical losses L_1 and L_2 are zero. Horizontal axis is optical phase ϕ. When optical phase ϕ is 0 or 180 degrees, the correlation variance $\Delta_{1,2}{}^2$ is greater than or equal to one and does not satisfy the inseparability criterion at each average photon number n. In these cases output beams are two independent single-mode squeezed light beams and therefore in separable states. When optical phase ϕ is controlled at around 90 degrees, the correlation variance $\Delta_{1,2}{}^2$ satisfies the inseparability criterion. So, the outputs are entangled two-mode squeezed light beams. It is noticeable that the correlation variances show so sensitive dependence on optical phase ϕ especially in the case with large average photon number n. Therefore the minute phase control

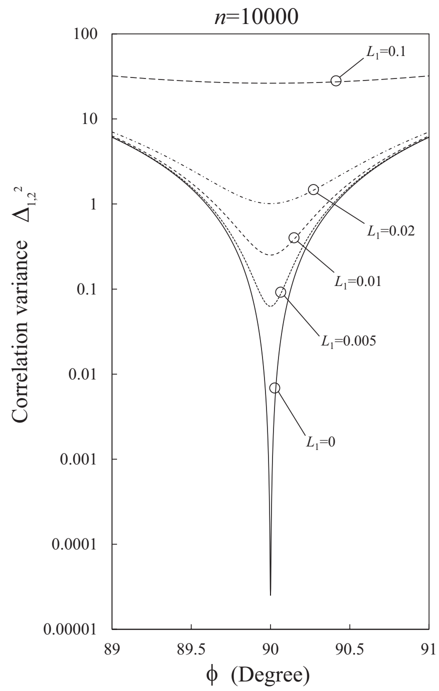

Fig. 5. Calculation results of correlaiton variance $\Delta_{1,2}{}^2$ with average phoon number n of 10000.

at 90 degrees is required to maintain the inseparability criterion in high-power two-mode squeezed light beams.

Fig. 4 shows accurate calculation results of correlation variance $\Delta_{1,2}{}^2$ at around 90 degrees of optical phase ϕ. (a) is a result of rather high-power two-mode squeezed light beams with a large number of photons ($n = 100$). (b) and (c) correspond to results with average photon number n of 10 and 1 which are comparable with current experimental progress of squeezed light generation [8]. In these calculations optical loss L_2 is set at zero and only the optical loss L_1 is considered as a variable parameter by assuming the quantum radar. Results show that the phase control within the angular range of 90±5.7, 90±12.5, and 90±32.8 degrees is required to maintain the inseparability criterion at each average photon number n of 100, 10, and 1 with optical loss L_1 of zero. Quantum entanglement shows rather fragile property especially in the case with large average photon number n of 100. The correlation variances show sensitive dependence on optical loss L_1 and satisfy the inseparability criterion with optical loss L_1 less than 0.2 as shown in Fig. 4 (a). It is noticeable that when optical loss L_1 is increased the angular range of the optical phase ϕ to maintain the inseparability criterion gradually decreases. On the other hand in the cases with rather small average photon number n of 10 and 1, the correlation variances are kept less than one up to 0.8 and 0.9 of optical loss L_1, respectively. Then the quantum entanglement shows much robust property against the optical loss.

Fig. 5 shows calculation results of correlation variance

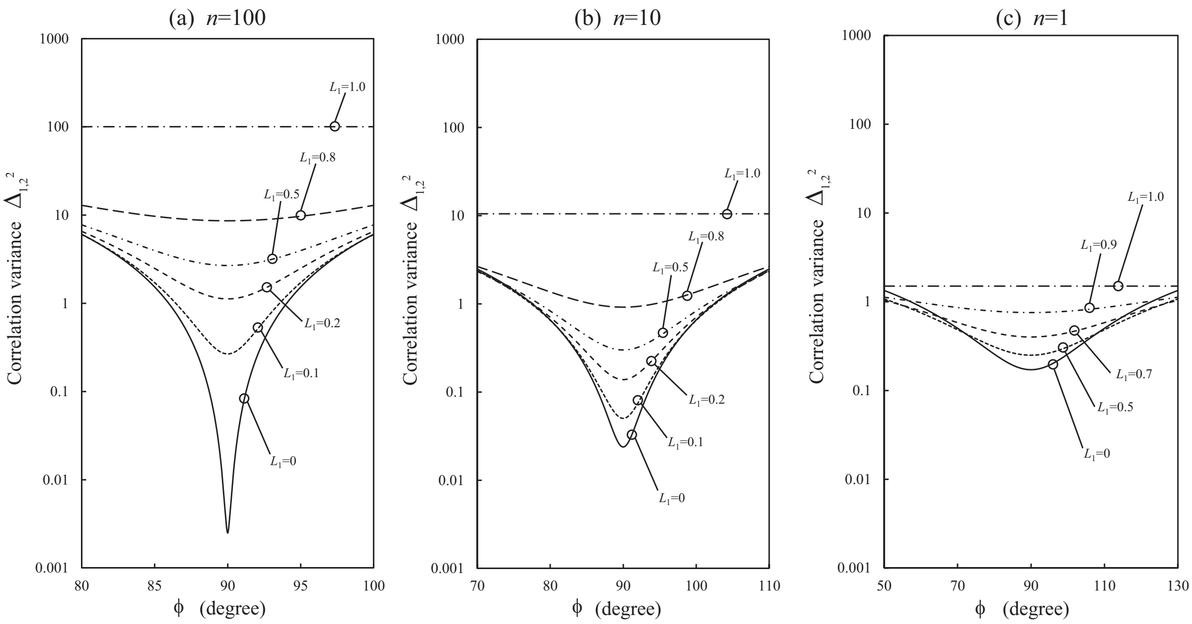

Fig. 4. Calculation results of correlaiton variance $\Delta_{1,2}{}^2$ with average phoon number n of (a)100, (b)10, and (c)1, respectively. In these calculations optical loss L_1 is varied and L_2 is set at zero by assuming the quantum radar.

$\Delta_{1,2}{}^2$ in the case of extremely high-power two-mode squeezed light source with average photon number n of 10000, althoufh it is impractical in current experimental progress. By assuming the quantum radar, optical loss L_1 is varied and L_2 is set at zero. The corrrelation variance $\Delta_{1,2}{}^2$ depends sensitively on optical phase ϕ. The phase control within the angular range of 90±0.41 degrees is required to maintain the inseparability criterion. The optical phase control with ±0.41 degrees seems to be feasible technology, since the optical phase control with ±0.66 is reported in the recent experiment of squeezed light generation [9]. The corrrelation variance $\Delta_{1,2}{}^2$ is also sensitive to the optical loss L_1 and satisfies the inseparability criterion less than 0.02 of L_1.

IV. Summary

The author is interested in applying a two-mode squeezed light source to quantum radar based on the quantum illumination method. Two-mode squeezed light can be generated by mixing two single-mode squeezed light beams by a beam splitter with controlling a relative optical phase at 90 degrees. In this article the correlation variances of quadrature phase amplitudes of two-mode squeezed light beams were derived as a function of relative optical phase ϕ and optical losses in outer environments. And quantum entanglement of the light source was analyzed based on Duan's and Simon's inseparability criterion. From an engineering perspective of quantum radar, the author showed how much minute control of optical phase ϕ is needed to maintain the quantum entanglement of the high-power two-mode squeezed light source. Calculation results show that optical phase control within the angular range of 90±0.41, 90±5.7, 90±12.5, and 90±32.8 degrees is required to maintain the inseparability criterion at each average photon number n of 10000, 100, 10, and 1. Results also show that the quantum entanglement of the two-mode squeezed light source is sensitively affected by optical losses in outer environments especially in the high-power cases.

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