Sensitivity of Optical-Phase Control in a High-Power Two-Mode Squeezed Light Source

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Abstract—Two-mode squeezed light is an important quantum entanglement resource for realizing quantum radar based on the quantum illumination method. It can be generated by mixing two single-mode squeezed light beams by a beam splitter with controlling a relative optical phase at 90 degrees. From an engineering perspective of quantum radar, it is important to evaluate how much minute control of the optical phase is needed at around 90 degrees of angular range, especially in the case of high-power two-mode squeezed light. Furthermore, it is also important to evaluate how much the controllability of the optical phase and quantum entanglement of the light source are affected by external environments. In the quantum radar system one of the entangled beams is transmitted through a turbulent atmosphere or a lossy optical medium. These environmental factors degrade quantum entanglement of the light source and affects performance of target detection.

In this article the author studied the controllability of optical phase of high-power two-mode squeezed light beams in a lossy optical medium which is one of the most probable examples of environmental factors. To verify quantum entanglement of the light source, an inseparability criterion is necessary. One useful and practical inseparability criterion for continuous-variable entangled states was developed by Duan, et al. [6] and Simon [7]. This criterion is based on a correlation variation of quadrature phase amplitudes of the light source which can be directly detected by balanced homodyne measurement. Therefore it has been frequently used in continuous-variable quantum-optics experiments to verify quantum entanglement [5]. In this article correlation variances of quadrature phase amplitudes of two-mode squeezed light beams are derived as a function of relative optical phase and optical losses in environments. Then the optical-phase controllability is discussed especially in the case of the high-power two-mode squeezed light source based on the inseparability criterion. The effects of optical losses on the optical-phase controllability of the light source are also discussed.

I. INTRODUCTION

Quantum illumination is a recently developed target detection method utilizing quantum entanglement and expected to be applied to quantum radar [1], [2], [3]. Initially a pair of entangled photons was proposed as a quantum entanglement resource [1]. One of the entangled photons is transmitted towards a target and reflected to a radar receiver. The other photon is directly sent to the receiver with a safe channel. The quantum correlation between entangled photons is exploited to improve error probability of discrimination for target presence or absence even in a lossy and noisy environment. Tan, et al. developed quantum illumination method with entangled two-mode Gaussian states such as two-mode squeezed light and showed improvement in error probability of target detection compared with a single-mode coherent state [2]. So, the author is interested in applying a two-mode squeezed light source to quantum radar based on the quantum illumination method [4].

Two-mode squeezed light is a macroscopic quantum entangled state of electro-magnetic fields and shows non-classical correlation between quadrature phase amplitudes in each optical beam. It is often generated by mixing two independent single-mode squeezed light beams using a beam splitter with transmissivity of 0.5. In previous experimental works, a relative optical phase between single-mode squeezed light beams was usually controlled at 90 degrees to generate quantum entanglement [5]. From an engineering perspective of quantum radar, it is important to evaluate how much minute control of the optical phase is needed at around 90 degrees of angular range, especially in the case of high-power two-mode squeezed light. Furthermore, it is also important to evaluate how much the controllability of the optical phase and quantum entanglement of the light source are affected by external environments. In the quantum radar system one of the entangled beams is transmitted through a turbulent atmosphere or a lossy optical medium. These environmental factors degrade quantum entanglement of the light source and affects performance of target detection.

II. OPTICAL MODEL

In this section a description of two-mode squeezed light is derived using Heisenberg representation (with unit of $\hbar = 1/2$, i.e. $[\hat{x}, \hat{y}] = i/2$) as is often used in continuous-variable quantum-optics experiments [5]. As shown in Fig. 1 entangled two-mode squeezed light beams are often generated by combining two single-mode squeezed light beams $SQ_1$ and $SQ_2$ with a beam splitter $BS(0.5)$ with transmissivity of 0.5. $\phi$ is a relative optical phase between squeezed light beams $SQ_1$ and $SQ_2$. 
It has been proven by Duan, et al. [6] and Simon [7] that two outputs Out1 and Out2 are inseparable and show quantum entanglement when the relation
\[ \Delta_{1,2}^2 < 1 \]  
is satisfied. Equation (6) is a sufficient condition of quantum entanglement and therefore called as an inseparability criterion. It seems that there is a relation between correlation variance \( \Delta_{1,2}^2 \) and quantum entanglement.
measure. However the quantitative relation between correlation variance $\Delta_{1,2}^2$ and the strength of quantum entanglement is not clear at this moment.

In our optical model, correlation variance $\Delta_{1,2}^2$ can be calculated using quadrature phase amplitudes of final output states Out$_1$ and Out$_2$ as

$$\Delta_{1,2}^2 = \frac{1}{4} \left\{ \frac{1}{2} (\sqrt{1 - L_1} + \sqrt{1 - L_2})^2 (1 + \sin^2 \phi) e^{-2r} + \frac{1}{2} (\sqrt{1 - L_1} - \sqrt{1 - L_2})^2 \cos^2 \phi e^{-2r} + \frac{1}{2} (\sqrt{1 - L_1} - \sqrt{1 - L_2})^2 (1 + \sin^2 \phi) e^{2r} + \frac{1}{2} (\sqrt{1 - L_1} + \sqrt{1 - L_2})^2 \cos^2 \phi e^{2r} \right\}$$

and are given as a function of optical phase $\phi$, squeezing parameter $r$ and optical losses $L_1$ and $L_2$.

III. CALCULATION RESULTS

Fig. 3 shows calculation results of correlation variance $\Delta_{1,2}^2$ with average photon number $n$ of 100, 30, 10, 3, 1, and 0 in the ideal case that both optical losses $L_1$ and $L_2$ are zero. Horizontal axis is optical phase $\phi$. When optical phase $\phi$ is 0 or 180 degrees, the correlation variance $\Delta_{1,2}^2$ is greater than or equal to one and does not satisfy the inseparability criterion at each average photon number $n$. In these cases output beams are two independent single-mode squeezed light beams and therefore in separable states. When optical phase $\phi$ is controlled at around 90 degrees, the correlation variance $\Delta_{1,2}^2$ satisfies the inseparability criterion. So, the outputs are entangled two-mode squeezed light beams. It is noticeable that the correlation variances show so sensitive dependence on optical phase $\phi$ especially in the case with large average photon number $n$. Therefore the minute phase control at 90 degrees is required to maintain the inseparability criterion in high-power two-mode squeezed light beams.

Fig. 4 shows accurate calculation results of correlation variance $\Delta_{1,2}^2$ at around 90 degrees of optical phase $\phi$. (a) is a result of rather high-power two-mode squeezed light beams with a large number of photons ($n = 100$). (b) and (c) correspond to results with average photon number $n$ of 10 and 1 which are comparable with current experimental progress of squeezed light generation [8]. In these calculations optical loss $L_2$ is set at zero and only the optical loss $L_1$ is considered as a variable parameter by assuming the quantum radar. Results show that the phase control within the angular range of 90±5.7, 90±12.5, and 90±32.8 degrees is required to maintain the inseparability criterion at each average photon number $n$ of 100, 10, and 1 with optical loss $L_1$ of zero. Quantum entanglement shows rather fragile property especially in the case with large average photon number $n$ of 100. The correlation variances show sensitive dependence on optical loss $L_1$ and satisfy the inseparability criterion with optical loss $L_1$ less than 0.2 as shown in Fig. 4 (a). It is noticeable that when optical loss $L_1$ is increased the angular range of the optical phase $\phi$ to maintain the inseparability criterion gradually decreases. On the other hand in the cases with rather small average photon number $n$ of 10 and 1, the correlation variances are kept less than one up to 0.8 and 0.9 of optical loss $L_1$, respectively. Then the quantum entanglement shows much robust property against the optical loss.

Fig. 5 shows calculation results of correlation variance $\Delta_{1,2}^2$ with average photon number $n$ of 10000.
\( \Delta_{1,2}^2 \) in the case of extremely high-power two-mode squeezed light source with average photon number \( n \) of 10000, although it is impractical in current experimental progress. By assuming the quantum radar, optical loss \( L_1 \) is varied and \( L_2 \) is set at zero. The correlation variance \( \Delta_{1,2}^2 \) depends sensitively on optical phase \( \phi \). The phase control within the angular range of \( 90 \pm 0.41 \) degrees is required to maintain the inseparability criterion. The optical phase control with \( \pm 0.41 \) degrees seems to be feasible technology, since the optical phase control with \( \pm 0.66 \) is reported in the recent experiment of squeezed light generation [9]. The correlation variance \( \Delta_{1,2}^2 \) is also sensitive to the optical loss \( L_1 \) and satisfies the inseparability criterion less than 0.02 of \( L_1 \).

IV. SUMMARY

The author is interested in applying a two-mode squeezed light source to quantum radar based on the quantum illumination method. Two-mode squeezed light can be generated by mixing two single-mode squeezed light beams by a beam splitter with controlling a relative optical phase at 90 degrees. In this article the correlation variances of quadrature phase amplitudes of two-mode squeezed light beams were derived as a function of relative optical phase \( \phi \) and optical losses in outer environments. And quantum entanglement of the light source was analyzed based on Duan’s and Simon’s inseparability criterion. From an engineering perspective of quantum radar, the author showed how much minute control of optical phase \( \phi \) is needed to maintain the quantum entanglement of the high-power two-mode squeezed light source. Calculation results show that optical phase control within the angular range of \( 90 \pm 0.41 \), \( 90 \pm 5.7 \), \( 90 \pm 12.5 \), and \( 90 \pm 32.8 \) degrees is required to maintain the inseparability criterion at each average photon number \( n \) of 10000, 100, 10, and 1. Results also show that the quantum entanglement of the two-mode squeezed light source is sensitively affected by optical losses in outer environments especially in the high-power cases.

REFERENCES