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On Several Errors Regarding Mathematics for Quantum Mechanics and Quantum Information Science

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***Abstract*—Mathematical errors tend to be propagated in the physics literature. Some have already been pointed out or mentioned in earlier volumes of this bulletin by the present author. In this note, another error found in a revised edition of a famous textbook on quantum mechanics is pointed out in view of the fact that the first edition of the book seems to have been standard and confusions may be likely to occur among the readers of the revised edition. It is on the name of a group consisting of matrices of some property. Some natures of this error and other errors this author has encountered are discussed.**

I. Introduction

Erroneous mathematical statements can sometimes be found in the physics literature, and they tend to be propagated in space and time. For example, in [1, p. 24], citing [2], the authors have stated that it is very common in the physics literature to find incomplete, or even incorrect usage of the Euler angles when half-integral angular momenta [SU(2)] are discussed. That work [2] pointed out an error regarding rotations and the present author gave a related result [3]. In [4], the present author has pointed out another error in textbooks on quantum computation (see also [5]). Noticing this error has led to a constructive result. Namely, a lemma [6, Lemma 6.1], which is originally found in an effort to disprove the erroneous statement, as explained in [4], has given rise to a constructive algorithm on rotations, which is the main result of [6].

The examples of errors mentioned above [2], [4], [5] are technical and do not seem ready to be correctable by all diligent readers. On the other hand, another error to be pointed out below is so simple that the reader having mathematical backgrounds may think the error to be trivial or the presentation below to be verbose. However, this would have, at least, pedagogical meaning; this note is not aimed at presenting original results.

J. J. Sakurai's 'Modern Quantum Mechanics' [7] would be one of the most approved introductory textbooks on quantum mechanics. At least, (its Japanese translation [9] of) this book became highly reputed among those learning physics soon after its publication when this author was an undergraduate student (majoring in applied mathematics but) receiving an education in an applied physics course. The aim of this note is to point out an error in later editions of this textbook in order that readers of them may not be confused. The error was made in the revised edition [8] of [7]. Note that the original author of [7] passed away before the first edition [7] was published. The fact that a more recent edition [10] with augmented authorship leaves this problem unnoticed or unattended makes this author feel the need for pointing it out.

II. Pointing Out An Error

First, a related terminology from mathematics is recalled. The group consisting of all 3×3 orthogonal matrices is called $\mathrm{O}(3)$, where O stands for orthogonal, and 3 for three dimensions. The group consisting of all 3×3 orthogonal matrices with *determinant* 1 is called $\mathrm{SO}(3)$, where S stands for special.

In the original edition [7], the term '$\mathrm{O}(3)$' has been introduced to discuss rotations, and the error to be pointed out is related to this part. Specifically, the author has stated [7], [8], in Section 3.3 thereof, that the set of all multiplication operations with orthogonal matrices forms a group, and that 'by this' he means that the following four requirements are satisfied:

1) The product of any two orthogonal matrices is another orthogonal matrix.
2) The associative law holds.
3) The identity matrix 1, which is defined by $R1 = 1R = R$, is a member of the class of all orthogonal matrices.
4) The inverse matrix R^{-1}, which is defined by $RR^{-1} = R^{-1}R = 1$, is also a member.

Right after this, the original edition [7] says that this group has the name $\mathrm{O}(3)$ and the revised edition [8], or a more recent edition [10], says that it has the name $\mathrm{SO}(3)$. Their description of the group in question does not change through the two editions. Hence, at least, one of the two statements is wrong.

This note points out the later edition [8] ([10]) is wrong. This is simply because what is described as a group above with requirements 1)–4) is the set of all 3×3 orthogonal matrices, namely, O(3).

III. Discussions

A. Why the Error Occurred

Guessing the reason why the error has occurred is a subjective act, so the following should be understood to include such a subjective view. A. Sakurai annotated, in his translation into Japanese [9], that the orthogonal group treated in that section is $\mathrm{SO}(3)$, which excludes space inversion, rather than $\mathrm{O}(3)$. (It seems that the first Japanese edition had this comment added after going through several printings: This comment can be found in the 8th printing [9] but not in the 2nd printing.) The present author guesses that this comment has been reflected in the revised English edition (1994) *imprecisely*. Namely, the name '$\mathrm{O}(3)$' was changed into '$\mathrm{SO}(3)$' without modifying the description of $\mathrm{O}(3)$. As a result, what is claimed to be $\mathrm{SO}(3)$, namely, the group specified with Eqs. (3.3.1) through (3.3.5) thereof remains $\mathrm{O}(3)$ in the later edition [8].[1]

B. More Technical Errors

The groups SU(2) and SO(3) are fundamental in diverse fields including mathematics and physics, and it is known that those two groups are closely related to each other. Recently, the author has obtained and discussed results on SU(2) and SO(3). The interested reader is referred to [6] for the relationship and results on SU(2) and SO(3).

Though the error pointed out in Section II would be simply an error in a description, technical errors that are not ready to be correctable by all diligent readers have often been found. Examples are already mentioned in the introduction [2], [4], [5]. In particular, the error treated in [4], [5] contradicts proven mathematical results [6].

C. Other Confusion

In a piece of work of the present author's [11], a sequence of polynomially constructible dual-containing geometric Goppa codes that attain the Tsfasman-Vladut-Zink (TVZ) bound was presented. Here, 'polynomially constructible' means 'constructible in polynomial time.' This issue was suggested in [12, Footnote 5]. The present author's motivation for solving this issue was to present polynomially constructible quantum error-correcting codes (QECCs), the main ingredient of which is a sequence of dual-containing geometric Goppa codes.

In this result, the level of technicality or specialty is much higher than the author's other results or comments on errors mentioned above. Though the technical details are not presented here because of such specialty, he mentions that the real incentive to solving this issue was this author's eagerness to clear up some confusion among those working with QECCs (many had seemed even ignorant of the problem to be explained briefly below), or simply, eagerness to know whether some codes of the desired properties exist or not. Namely, at the time of tackling this issue, it was not known whether a sequence of *polynomially constructible* dual-containing geometric Goppa codes that attain the TVZ bound exist or not. From Ashikhmin *et al.*'s paper on QECCs [13], it can be seen that if such a sequence of codes exist, then it can be used to construct quantum error-correcting codes that improves on QECCs of Ashikhmin *et al.*'s [13], replacing the main ingredient, which is a sequence of dual-containing geometric Goppa codes attaining some bound *smaller* than the TVZ bound, by the *polynomially constructible* geometric Goppa codes attaining the TVZ bound.

Some, without solving this issue, proposed and compared QECCs that were not known to be polynomially constructible (or proven so, if valid) with Ashikhmin *et al.*'s QECCs, alleging an improvement over Ashikhmin *et al.*'s QECCs. This was clearly unfaithful to our principles of researchers (at least, for this author) since it is a rule that comparisons should be made among things (codes constructed, in this case) under the same conditions (requirements), whereas the alleged improvement refers to that codes *unproven* to be polynomially constructible codes outperform codes *proven* to be polynomially constructible in terms of the standard measure of performance in coding theory. (For the technical details, the reader is referred to [12, Footnote 5] and [11].) Thus, noticing unsoundness, specifically, that a basic rule was violated, was an incentive to that work [11].

D. Coping With Errors

It seems careful readings are required more and more for the physics literature. Then, how can we protect ourselves from such errors? In the specific example treated in Section II, the power of logic can be recognized: Noticing the description of the group in question is the same among the two editions, one is ready to conclude that, at most, only one of SO(3) and O(3) is qualified to be the name of the group even if the reader may encounter the terms SO(3) and O(3) for the first time provided that he/she judges SO(3) and O(3) to be different from each other. Hence, this error is simple enough to be spotted.

[1] In this author's opinion, the scope that A. Sakurai's annotation [9] applies to can be limited to pp. 171–174 of [7, Section 3.3]. Specifically, A. Sakurai's annotation [9] can be accommodated as follows, where this should be understood to be for the English edition [7]: The orthogonal group treated in the first paragraph on p. 171 (rather than in that whole section) is $\mathrm{SO}(3)$, which excludes space inversion, rather than $\mathrm{O}(3)$. Therefore, a minimum correction to [7] would have been to alter '$\mathrm{O}(3)$' into '$\mathrm{SO}(3)$' in the first paragraph on p. 171, namely, only after Eq. (3.3.14) in Section 3.3 of [7] (in the five places), and maybe in the section title in addition. In this case, the difference between SO(3) and O(3) should be mentioned. This can be done, for example, in the paragraph just before the paragraph in question (where the difference between SU(2) and U(2) is described).

In general, finding contradictions would be one of the easiest ways to find errors.

The examples treated or mentioned in Sections II and III-B are essentially logical in that it can be clarified with logical arguments in standard manners. An incident mentioned in Section III-C is related to the principles of researchers, which this author has learned at elementary school if he remembers correctly. Therefore, the least level of education and compliance to right rules would suffice in theory. In practice, that incident happened unfortunately. The author hopes that no such unsound incidents occur any more, which would be possible by the adherence to our principles.

The above incidents, of finding and correcting errors or settling some confusion, that this author was involved with would be all logical, theoretical or mathematical, and the activities in these incidents might be said to be closed in human thoughts. Some may be interested in errors that occur in the process of perception. See, e.g., [14, pp. 108–125].[2]

Incidentally, (as he has conducted researches on security issues, which are independent of those activities of his colleagues, or most of vague assertions found in the literature, that may be related to topics on security) this author also comments that any proof of security of a cryptographic system or protocol, whether it exploits quantum theoretical features of devices or not, should be logical and mathematical, so that the logical way of thinking, which has been sometimes emphasized in this and earlier articles of this author in this bulletin, would be important in treating cryptographic issues. (In discussing the security of some system, whether premises on which a security proof is based are appropriate or not is another, probably, more, important thing. But it can be separated from a proof of the security.)

IV. Summary

This note has pointed out an error in a revised edition of a textbook in order that readers of it may not be confused. Specifically, in Section 3.3 of the famous book [7], the first ed., it says that the set of all multiplication operations with orthogonal matrices forms a group, and in particular, that this group (consisting of all 3×3 orthogonal matrices) has the name $\mathrm{O}(3)$. In the revised edition [8], it wrongly says that this group has the name $\mathrm{SO}(3)$. The importance of logic in coping with wrong arguments has been discussed.

References

[1] L. C. Biedenharn and J. D. Louck, *Angular momentum in quantum physics: theory and application*. Reading, Mass., United States: Addison-Wesley, 1981.

[2] J. E. Jonker and E. D. Vries, “A note on orthogonality and completeness of the rotation matrices,” *Nuclear Physics A*, vol. 105, no. 2, pp. 621–626, 1967.

[3] M. Hamada, “On parametrizations for rotations,” *Tamagawa University Quantum ICT Research Institute Bulletin*, vol. 5, pp. 25–28, Dec. 2015.

[4] M. Hamada, “A lemma on Euler angles,” *Tamagawa University Quantum ICT Research Institute Bulletin*, vol. 3, pp. 25–27, Dec. 2013.

[5] M. Hamada, “A simple demonstration of a fallacy in implementability arguments on quantum computation,” *Tamagawa University Quantum ICT Research Institute Bulletin*, vol. 4, pp. 31–32, Dec. 2014.

[6] M. Hamada, “The minimum number of rotations about two axes for constructing an arbitrarily fixed rotation,” *Royal Society Open Science*, vol. 1, p. 140145, Nov. 2014. http://dx.doi.org/10.1098/rsos.140145.

[7] J. J. Sakurai, *Modern Quantum Mechanics*. Menlo Park: Benjamin/Cummings Publishing, 1985.

[8] J. J. Sakurai, *Modern Quantum Mechanics*. Menlo Park: Benjamin/Cummings Publishing, Revised ed., 1994.

[9] J. J. Sakurai, *Modern Quantum Mechanics (Japanese Translation)*. Kyoto: Yoshioka-Shoten, 8th printing, 1997. (The 1st printing was published in 1989.)

[10] J. J. Sakurai and J. Napolitano, *Modern Quantum Mechanics*. San Francisco: Addison-Wesley, 2nd ed., 2011.

[11] M. Hamada, “A polynomial-time construction of self-orthogonal codes and applications to quantum error correction,” *Proc. IEEE International Symposium on Information Theory (ISIT)*, pp. 794–798, 2009.

[12] M. Hamada, “Concatenated quantum codes constructible in polynomial time: Efficient decoding and error correction,” *IEEE Trans. Information Theory*, vol. 54, pp. 5689–5704, Dec. 2008.

[13] A. E. Ashikhmin, S. Litsyn, and M. A. Tsfasman, “Asymptotically good quantum codes,” *Phys. Rev. A*, vol. 63, pp. 032311–1–5, 2001.

[14] E. Mach, *Erkenntniss und Irrtum. Skizzen zur Psychologie der Forschung*. Leipzig: Verlag von Johann Ambrosius Barth, 1920.

[15] H. Yukawa and T. Inoue, *Gendai no Kagaku I (Modern Science I)*. Tokyo: Chuoukouronsha, 1973. In Japanese.

[2] A Japanese translation of the indicated part is included as ‘Ninshiki to Gobyu’ in [15, pp. 395–408].