Quantum Steerability for Entangled Coherent States

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We study a stronger form of quantum entanglement, i.e. *quantum steerability*, of entangled coherent states (ECS). We find that the criterion employing entropic uncertainty relation can reveal quantum steering for ECS of any size whereas Reid criterion using Heisenberg uncertainty relation scarcely can. The steering behavior of the state is examined when it is subject to a certain noisy environment. We also show that when asymmetric decoherence arises—e.g., only one part of bipartite system experiences noise—steering is possible only in one direction.

I. INTRODUCTION

Quantum entanglement and nonlocality are not only of fundamental importance in their own rights but indispensable resources in various quantum information tasks [1, 2]. Quantum steering lies between these two categories of nonlocal correlations which cannot be explained by any realistic theory. Historically, the notion of *steering* was introduced in Schrödinger's paper [3] which was intended to respond to the arguments presented by Einstein, Podolsky, and Rosen (EPR) [4]. In their paper, EPR argued that quantum mechanics lacks physical *reality* in its elements and consequently is incomplete. Schrödinger later named the variance between classical and quantum mechanics as Einstein-Podolsky-Rosen (EPR) paradox [3].

M. Reid first attempted to examine EPR paradox with now so called *steering* scenario [5]. She studied the entanglement of continuous-variable (CV) entangled states as originally discussed by EPR and came up with a criterion based on inferred uncertainty relation. Subsequently, there have been experiments to demonstrate steering [6–10]. Wiseman *et al.* later clarified that steering can be regarded as an information task to prove the existence of entanglement between two parties [11]. Later authors generalized CV Reid criterion to discrete system [12] and another authors also devised a criterion using entropy [9].

One of the salient features of steering different from the notions of entanglement and nonlocality is *asymmetry*: verification of steering depends on an asymmetric model. When it is required to verify the existence of entanglement or nonlocality of a state, one assumes beforehand the possibility that it can be described—when measured by local observables of two parties—by a joint probability of measurement outcomes, which is determined by a *hidden variable*. Such joint probability models are often referred to as *quantum separable state* model and *local hidden variable* model respectively. Likewise, in the case of steering, a *non-steerable* model is presumed, which can also be addressed by a hidden variable. In this model, however, one party's statistical property is differently assumed from the other while in the former two models those properties of the two parties are assumed to be of the same kind—this will be clarified soon. Because of this asymmetry inherent in the steering model, there has also been studies on unidirectional steering [13, 14].

In this article, we will consider a certain class of non-Gaussian CV states, namely, entangled coherent states (ECS). It is considered one of leading candidates for future quantum information processing [15]. While its characteristics of entanglement and nonlocality is well studied [16, 17], it is not yet investigated from the perspective of quantum steering to our knowledge.

In the ket notation, ECS is written as

$$|\Phi_\pm(\alpha,\beta)\rangle = \mathcal{N}_\pm(\alpha,\beta)\left(|\alpha\rangle_A\,|\beta\rangle_B \pm |-\alpha\rangle_A\,|-\beta\rangle_B\right), \quad (1)$$

where α and β are assumed to be real and $\mathcal{N}_\pm(\alpha,\beta) = \left(2 \pm 2e^{-2\alpha^2-2\beta^2}\right)^{-1/2}$ is the normalization factor. Since Φ_+ (Φ_-) has only even (odd) sums of number of particles, it is often referred to even (odd) ECS. Furthermore, when $\beta = \alpha$ it becomes symmetric with respect to each party.

II. REID CRITERION

Wiseman *et al.* rigorously formulated the concept of steering as an information task [11] to verify entanglement in the following scenario. Alice wants to convince Bob that she sends one part of an entangled state but Bob does not trust Alice and assumes that there is a some kind of mapping and ensemble of states which are hidden to him. However, despite the possibility of Alice's cheating him, he believes that his system is described by quantum mechanics and hence considers so called a local hidden state (LHS) model. In this model, the joint probability of obtaining measurement outcomes a and b by the two parties respectively can be written as

$$P(a,b) = \sum_\lambda P(\lambda)P(a|\lambda)P_Q(b|\lambda), \quad (2)$$

where $P(\lambda)$ is the distribution of a hidden variable λ and the subscript Q implies that Bob's part of particles is governed by quantum mechanics. Although Alice cannot affect the Bob's local measurement result

$$P(b) = \sum_a P(a,b) = \sum_\lambda P(\lambda)P_Q(b|\lambda) \quad (3)$$

—otherwise it would violate no-signaling condition—she might prepare states belonging to a particular ensemble with probability $P(a) = \sum_b P(a,b)$ by measuring her own state and obtaining outcome a. However, Bob is suspicious if Alice would possibly deceive him using her knowledge of a stochastic map $P(a|\lambda)$ from λ to a which is unknown to Bob. Accordingly the probability of Bob's getting outcome b conditioned on Alice's outcome a is given by

$$P(b|a) = P(a,b)/P(a) = \sum_\lambda \frac{P(\lambda)P(a|\lambda)}{P(a)} P_Q(b|\lambda). \quad (4)$$

If Bob fails to find any such map and consequently his measurement result cannot be explained by the above LHS model, he must admit that the shared state is truly entangled.

One may want to compare this LHS model with the local hidden variable (LHV) model which can be used to refute a claim that the shared state are entangled when a third party, say Charlie, does not trust both Alice and Bob. In this case, one should assume the following form of joint probability

$$P(a,b) = \sum_\lambda P(\lambda)P(a|\lambda)P(b|\lambda), \quad (5)$$

which is also described by a hidden variable λ.

As another extreme case, if two parties Alice and Bob are both trustable, Charlie can request them to falsify the following quantum separable state model

$$P(a,b) = \sum_\lambda P(\lambda)P_Q(a|\lambda)P_Q(b|\lambda) \quad (6)$$

in order to show the existence of entanglement. It must be noted that since the probability distribution $P(b|\lambda)$ without a subscript is not constrained by quantum statistics it is harder to falsify Eq. (2) than Eq. (6); by the same reasoning it is harder to falsify Eq. (5) than Eq. (2). Therefore, the hierarchy among these three categories of nonlocal correlations follows as

nonlocality $\Longrightarrow$ steerability $\Longrightarrow$ entanglement.

As can be inferred from this relation, there exist entangled states that are not steerable and similarly steerable states that are not nonlocal. Indeed, many examples showing this hierarchy have been reported [8, 11, 12, 18].

The original Reid criterion is based on Heisenberg uncertainty relation of conditioned probability distribution functions (PDFs) of position and momentum [5]. Those PDFs of a generic quantum state ρ can be obtained using wave functions of a coherent state $|\alpha\rangle$ in terms of quadrature variables x and p

$$\langle x|\alpha\rangle = \frac{1}{\sqrt[4]{\pi}} e^{-\frac{1}{2}x^2+\sqrt{2}\alpha x-\alpha^2}, \quad \langle p|\alpha\rangle = \frac{1}{\sqrt[4]{\pi}} e^{-\frac{1}{2}p^2-i\sqrt{2}\alpha p}. \quad (7)$$

The position PDF (xPDF), its marginal and conditional PDFs are easily calculated respectively

$$P(x_A, x_B) = \langle x_A, x_B|\,\rho\,|x_A, x_B\rangle, \quad (8)$$

$$P(x_A) = \int dx_B P(x_A, x_B), \quad (9)$$

$$P(x_B|x_A) = P(x_A, x_B)/P(x_A), \quad (10)$$

and the momentum PDFs (pPDFs) are given in the same fashion. Or, alternatively, if the Wigner function of a state is given, the above PDFs can also be obtained from it

$$P(x_A, x_B) = \iint dp_A dp_B W(x_A, p_A, x_B, p_B), \quad (11)$$

$$P(p_A, p_B) = \iint dx_A dx_B W(x_A, p_A, x_B, p_B). \quad (12)$$

From these PDFs, the *inferred* variance of, say x, is given by

$$\begin{aligned}\Delta^2_{\rm inf}(X_B|X_A) &\equiv \int dx_A P(x_A)\, \Delta^2_{\rm inf}(X_B|X_A = x_A) \\ &= \int dx_A P(x_A) \int dx_B P(x_B|x_A)(x_B - m_{\rm est})^2. \quad (13)\end{aligned}$$

Note that the above variance is minimized when $m_{\rm est}$ is the conditioned mean:

$$m_{\rm est} = \langle X_B\rangle_A \equiv \int dx_B P(x_B|x_A)x_B. \quad (14)$$

The variance $\Delta^2_{\rm inf}(P_B|P_A)$ can be obtained in the same manner. With these two variances given, Reid inequality (RI) consequently reads

$$\Delta_{\rm inf}(X_B|X_A)\,\Delta_{\rm inf}(P_B|P_A) \geq \frac{1}{2}. \quad (15)$$

Reid criterion has been successful in some cases such as Gaussian states, while, on the other hand, it is not always a sufficiently good criterion; we will see one of these cases when applying it to ECS.

One can easily get the wave functions of ECSs using Eq. (7) as

$$\begin{aligned}\langle x_A, x_B|\Phi_\pm(\alpha,\beta)\rangle = \frac{\mathcal{N}_\pm(\alpha,\beta)}{\sqrt{\pi}}\Big[&e^{-\frac{1}{2}\left(x_A-\sqrt{2}\alpha\right)^2-\frac{1}{2}\left(x_B-\sqrt{2}\beta\right)^2} \\ &\pm e^{-\frac{1}{2}\left(x_A+\sqrt{2}\alpha\right)^2-\frac{1}{2}\left(x_B+\sqrt{2}\beta\right)^2}\Big], \quad (16)\end{aligned}$$

$$\begin{aligned}\langle p_A, p_B|\Phi_\pm(\alpha,\beta)\rangle = \frac{\mathcal{N}_\pm(\alpha,\beta)}{\sqrt{\pi}} e^{-\alpha^2-\beta^2}\Big[&e^{-\frac{1}{2}\left(p_A+i\sqrt{2}\alpha\right)^2-\frac{1}{2}\left(p_B+i\sqrt{2}\beta\right)^2} \\ &\pm e^{-\frac{1}{2}\left(p_A-i\sqrt{2}\alpha\right)^2-\frac{1}{2}\left(p_B-i\sqrt{2}\beta\right)^2}\Big], \quad (17)\end{aligned}$$

and subsequently their joint PDFs as

$$\begin{aligned}P_{\Phi_\pm}(x_A, x_B) &= |\langle x_A, x_B|\Phi_\pm(\alpha,\beta)\rangle|^2, \\ P_{\Phi_\pm}(p_A, p_B) &= |\langle p_A, p_B|\Phi_\pm(\alpha,\beta)\rangle|^2. \quad (18)\end{aligned}$$

The variances $\Delta_{\rm inf}(X_B|X_A)$, $\Delta_{\rm inf}(P_B|P_A)$ for the symmetric ECSs ($\beta = \alpha$), i.e., $\Phi_\pm(\alpha,\alpha)$ can be easily evaluated by the above probabilities and their products are plotted in Fig. 1(a). One can find that RI (15) is violated only in a narrow and restricted region for $|\Phi_+(\alpha,\alpha)\rangle$ while it is never violated for $|\Phi_-(\alpha,\alpha)\rangle$. Thus, the violation of RI seems to depend on the size and parity of ECSs.

It is worth commenting that since Alice's measurement observables can be arbitrary ones—when constructing a LHS model there is no constraint on Alice's observables—one may have another form of RI, e.g.,

$$\Delta_{\rm inf}(X_B|P_A)\Delta_{\rm inf}(P_B|X_A) \geq \frac{1}{2}, \quad (19)$$

which, however, does not lead to any violation for ECSs.

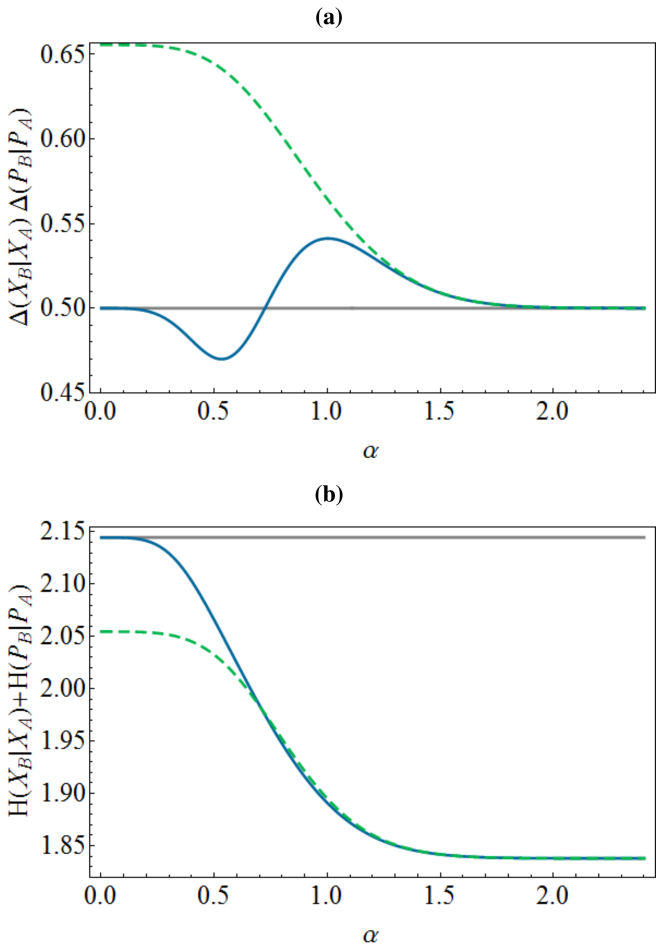

FIG. 1. (Color online) (a) The products of inferred uncertainties and (b) the sum of inferred Shannon entropies of ECSs $|\Phi_{\pm}(\alpha,\alpha)\rangle$ plotted against the magnitude α. In each panel, the two curves represent the cases of $|\Phi_{+}(\alpha,\alpha)\rangle$ (blue solid) and $|\Phi_{-}(\alpha,\alpha)\rangle$ (green dashed) respectively, and horizontal (gray solid) lines indicate the corresponding uncertainty bounds. Whereas the violation of RI is confined to a limited region of amplitude α only for the even parity ($|\Phi_{+}\rangle$), ESI is violated without any restriction on the amplitude and parity.

III. ENTROPIC STEERING CRITERION

As learned in the previous section, RI is not so efficient as to detect the steerability of non-Gaussian states such as ECSs. It is because RI depends only on the second order of observables; it is sufficiently good for a Gaussian state whose non-classical property depends solely on its second-order moments of position and momentum whereas it is not the case for a non-Gaussian state whose properties also depend on its higher-order moments. Therefore, we will try another steering inequality by Walborn *et al.* [9], which is based on *entropic* uncertainty relation (EUR) [19]

$$H(X) + H(P) \geq \ln(e\pi), \tag{20}$$

where

$$H(X) \equiv H[P(x)] = -\int dx P(x)\ln P(x) \tag{21}$$

is the Shannon entropy for xPDF; $H(P)$ is that for pPDF defined in the same manner. Equipped with these entropic measures of uncertainty other than the variance, the inferred entropy for xPDF can be defined in a similar fashion as the inferred variances are defined,

$$H(X_B|X_A) \equiv \int dx_A P(x_A) H(X_B|x_A), \tag{22}$$

$$\begin{aligned} H(X_B|x_A) &\equiv H[P(x_B|x_A)] \\ &= -\int dx_B P(x_B|x_A)\ln P(x_B|x_A), \end{aligned} \tag{23}$$

and $H(P_B|P_A)$ is defined in a similar way. Here, the conditional probability

$$P(x_B|x_A) = \sum_{\lambda} P(\lambda|x_A)P(x_B|\lambda) \tag{24}$$

is evaluated by Eq. (4) under the assumption of LHS modeling; here, the probability $P(\lambda|x_A)$ is short for $P(\lambda)P(x_A|\lambda)/P(x_A)$. Then, applying the concavity of Shannon entropy to the above convex-sum representation of the conditional probability, we get

$$\begin{aligned} H(X_B|x_A) &= H\left[\sum_{\lambda} P(\lambda|x_A)P(x_B|\lambda)\right] \\ &\geq \sum_{\lambda} P(\lambda|x_A) H[P(X_B|\lambda)], \end{aligned} \tag{25}$$

and accordingly by Eq. (22)

$$H(X_B|X_A) \geq \sum_{\lambda} P(\lambda) H[P(X_B|\lambda)], \tag{26}$$

A similar inequality can be obtained for pPDF. Finally, applying EUR (20) to each hidden state leads to the desired entropic steering inequality (ESI)

$$H(X_B|X_A) + H(P_B|P_A) \geq \ln(e\pi). \tag{27}$$

We apply this ESI to ECSs and find that the above inequality is violated by ECSs with any parity—even and odd—and for any amplitude; see Fig. 1(b). Therefore, one can say that ESI can fully detect the steerability of ECSs.

Again, one may also consider an observable-crossed form of ESI as in Eq. (19), e.g.,

$$H(X_B|P_A) + H(P_B|X_A) \geq \ln(e\pi). \tag{28}$$

However, this does not lead to any violation either, hence can be regarded as useless in this work.

Let us close this section by mentioning the case of applying ESI to Gaussian states. Since position-momentum EUR is equivalent to position-momentum HUR for the Gaussian states, position-momentum ESI also gives the same result as RI does for Gaussian states.

IV. STEERING UNDER NOISY ENVIRONMENT

When implementing a real experimental setup to detect the steerability of quantum states, the effect of noisy environment is unavoidable, which accordingly degrades the purity—and presumably also steerability. In order to consider the effect of mixedness due to noise, we introduce a specific noisy channel, namely, amplitude damping channel (ADC) under which the average particle number of a quantum state monotonically decreases with time. In the regime of Born-Markov approximation, zero temperature reservoir is often modeled by ADC and it causes a quantum state to lose its energy and eventually lead to the vacuum state. This vacuum-environment decoherence model is often described by the following master equation

$$\frac{d\rho}{d\tau} = \mathcal{L}(\hat{a})[\rho] + \mathcal{L}(\hat{b})[\rho], \qquad (29)$$

where τ = (decay rate)×(time) is the dimensionless time and $\mathcal{L}(\hat{o})[\rho]$ is the generic superoperator in the Lindblad form

$$\mathcal{L}(\hat{o})[\rho] = \hat{o}\rho\hat{o}^\dagger - \frac{1}{2}\rho\hat{o}^\dagger\hat{o} - \frac{1}{2}\hat{o}^\dagger\hat{o}\rho. \qquad (30)$$

From here on, we will refer to Eq. (29) as two-mode ADC to avoid possible confusion with another decoherence scenario later. The solution of the above master equation is well known and can be solved in many ways; here we will particularly adopt Krauss-opertor technique [20]:

$$\rho(\tau) = \sum_{n=0}^{\infty} \hat{K}_n(\hat{a})\hat{K}_n(\hat{b})\,\rho(0)\hat{K}_n^\dagger(\hat{a})\hat{K}_n^\dagger(\hat{b}) \qquad (31)$$

with the relevant Krauss operator

$$\hat{K}_n(\hat{o}) = \frac{r^n}{\sqrt{n!}} t^{\hat{o}^\dagger\hat{o}}\hat{o}^n \quad (t = e^{-\tau/2},\ r = \sqrt{1-t^2}). \qquad (32)$$

Note that $r = \sqrt{1-e^{-\tau}}$ can be regarded as a *normalized* time since $r(\tau = 0) = 0$ and $r(\tau = \infty) = 1$; hereafter, we will use r instead of τ when plotting entropic functions against time.

Using the singe-mode version of Eq. (31) or Eq. (38) one can easily find that normalized coherent-state projection simply transforms under ADC as

$$\frac{|\alpha\rangle\langle\beta|}{\langle\beta|\alpha\rangle} \xrightarrow{\text{ADC}} \frac{|t\alpha\rangle\langle t\beta|}{\langle t\beta|t\alpha\rangle}. \qquad (33)$$

Using this formula, one can obtain density matrices of ECSs suffering ADC

$$\begin{aligned} &\rho_{\mathrm{ECS}_\pm}(0) = |\Phi_\pm(\alpha,\beta)\rangle\langle\Phi_\pm(\alpha,\beta)| \\ \xrightarrow{\text{ADC}}\ &\rho_{\mathrm{ECS}_\pm}(\tau) = F_\pm\,|\Phi_\pm(t\alpha,t\beta)\rangle\langle\Phi_\pm(t\alpha,t\beta)| \\ &\qquad + (1-F_\pm)\,|\Phi_\mp(t\alpha,t\beta)\rangle\langle\Phi_\mp(t\alpha,t\beta)|, \end{aligned} \qquad (34)$$

where

$$F_\pm = \left[\frac{\mathcal{N}_\pm(\alpha,\beta)}{\mathcal{N}_\pm(t\alpha,t\beta)}\right]^2 \frac{1+e^{-2r^2(\alpha^2+\beta^2)}}{2}. \qquad (35)$$

Notice that as time elapses, $\rho_{\mathrm{ECS}_\pm}(\tau)$ eventually approaches two-mode vacuum state $|00\rangle_{AB}$ since $F_\pm \to (1\pm 1)/2$ as $\tau \to \infty$. Using these density matrices of ECSs, one can easily get joint xPDFs

$$P_{\mathrm{ECS}_\pm}(x_A,x_B) = F_\pm P_{\Phi_\pm}(x_A,x_B) + (1-F_\pm)P_{\Phi_\mp}(x_A,x_B), \qquad (36)$$

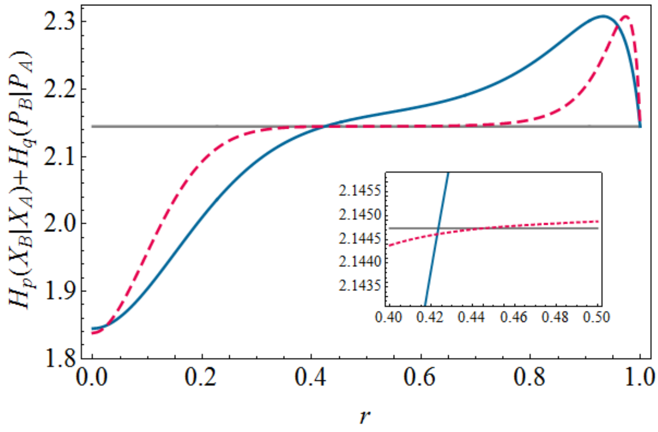

FIG. 2. (Color online) The sum of inferred entropies of xPDF and pPDF for ECSs plotted against the normalized time r for $\langle\hat{n}(0)\rangle = 4$ (blue solid) and $\langle\hat{n}(0)\rangle = 10$ (red dashed). The horizontal (gray solid) line indicates the entropic uncertainty bound $\ln(e\pi)$. The inset shows a magnified region where the violation—equivalently steerability—starts to disappear. See the main text for the detailed violation time of ESI.

and in the same way pPDFs.

For the case of pure ECS, entropic criterion is shown (in the previous section) to be better than Reid criterion. We next study the robustness of the steerability when ECS is subject to noise. We plot in Fig. 2(a) the sum of inferred entropies for xPDF and pPDF against the normalized time r. We have tested the cases that the states have initial average particle numbers $\langle\hat{n}(0)\rangle = \langle\hat{n}_A(0) + \hat{n}_B(0)\rangle = 4$ and 10 where $\hat{n}_A = \hat{a}^\dagger\hat{a}$ and $\hat{n}_B = \hat{b}^\dagger\hat{b}$ are the local number operators. Note that only one parity of $\rho_{\mathrm{ECS}_\pm}(\tau)$ is plotted since even and odd ECSs do not give any appreciable difference for such large amplitudes. The steerability is observed until r reaches 0.42 when $\langle\hat{n}(0)\rangle = 4$ while it is until r reaches 0.44 when $\langle\hat{n}(0)\rangle = 10$. Apparently, the larger-size ECS turns out to endure the noise longer than the smaller-size one. However, as one can see in the figure, the difference is rather slight and from the perspective of detectability, it seems to be the other way around, i.e., the smaller-size one is better detectable until steering is possible.

V. ONE-WAY STEERING

Since the steering scenario is born to be asymmetric as mentioned earlier, there is possibility to steer in one direction but not in the other. Recently, this feature has been given attention to by some authors [13, 14] and experimentally demonstrated [14].

In Ref. [14], the authors prepared an entangled state by beam-splitting a single-mode squeezed state and making only one (say, Bob's) mode mixed with the vacuum noise. Using this asymmetric state, they found a so called *one-way* steering condition where Alice can steer Bob's state but not the other way around. As a matter of fact, the effect of ADC is equivalent to mixing a given state with vacuum noise through a beam

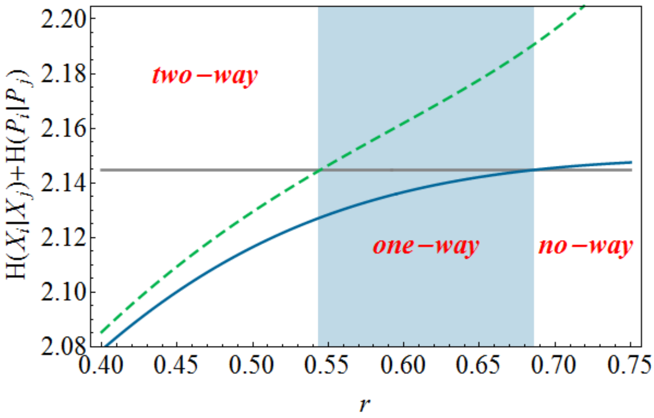

FIG. 3. (Color online) The sum of inferred entropies for xPDF and pPDF of ECS when $\langle\hat{n}(0)\rangle = 4$. The solid curves denote the case when Alice steers Bob's state ($i = B$, $j = A$) and the dashed ones the other way around ($i = A$, $j = B$). The horizontal (gray solid) lines indicate the corresponding entropic uncertainty bounds. The term "two-way" means that both Alice and Bob can steer each other's state; the term "one-way" in the colored region that Alice can steer but Bob cannot; finally, "no-way" that no one can steer.

splitter. Thus, by the same asymmetrization method, we can also test this possibility of one-way steering for ECS: we feed only one mode of the bipartite system into ADC. Here, we assume that Bob's mode undergoes ADC.

In order for Bob's mode to go under ADC, the relevant master equation should change from Eq. (29) to a *single-mode* ADC

$$\frac{d\rho}{d\tau} = \mathcal{L}(\hat{b})[\rho]. \tag{37}$$

Accordingly, its solution also changes from Eq. (31) to

$$\rho(\tau) = \sum_{n=0}^{\infty} \hat{K}_n(\hat{b})\,\rho(0)\hat{K}_n^{\dagger}(\hat{b}) \tag{38}$$

with the same Krauss operator as in Eq. (32).

So far, since we have dealt with symmetric states, we need only to consider the case that Alice steers Bob's state and that he examines her ability to steer. Now, if Bob attempts to steer Alice's state, she should use Eq. (27) with subscripts A and B interchanged, i.e., the following ESI

$$H(X_A|X_B) + H(P_A|P_B) \geq \ln(e\pi). \tag{39}$$

Under the single-mode ADC given in Eq. (37), the initially symmetric ECSs $|\Phi_\pm(\alpha,\alpha)\rangle\langle\Phi_\pm(\alpha,\alpha)|$ evolve now in an asymmetric fashion as

$$\begin{aligned}\rho'_{\mathrm{ECS}_\pm}(\tau) = {} & F'_\pm\,|\Phi_\pm(\alpha,t\alpha)\rangle\langle\Phi_\pm(\alpha,t\alpha)| \\ & + (1-F'_\pm)\,|\Phi_\mp(\alpha,t\alpha)\rangle\langle\Phi_\mp(\alpha,t\alpha)|\end{aligned} \tag{40}$$

with

$$F'_\pm = \left[\frac{\mathcal{N}_\pm(\alpha,\alpha)}{\mathcal{N}_\pm(\alpha,t\alpha)}\right]^2 \frac{1+e^{-2r^2\alpha^2}}{2}. \tag{41}$$

In the same manner as in the case of two-mode (symmetric) ADC, we can probe the above ECS by applying now two ESIs (27) and (39). In Fig. 3(a), we plot the sum of entropies of ECS of $\langle\hat{n}(0)\rangle = 4$ for the two circumstances: Alice steers Bob and vice versa. It is seen that Alice can steer Bob's state until $r = 0.68$ while he can until $r = 0.54$. Therefore, one can say that the colored region ($0.54 \leq r \leq 0.68$) in the figure indicates one-way steering condition for ECS.

At this point, it is worthy of notice that in one-way steering scenario a less decohered party (here, Alice) can happen to steer the other party (Bob) but that the reverse situation—i.e., more decohered party steer the other—does not happen. It becomes more convincing from the observation that the direction of one-way steering remains the same as the size of ECS changes. For instance, for the case of a larger initial size $\langle\hat{n}(0)\rangle = 10$, only the steering region of each party extends a bit more—the steering regions move forward from $r = 0.54$ to 0.57 for Alice and 0.68 to 0.70 for Bob—but the direction of steering does not change.

VI. CONCLUDING REMARKS

We investigated the steerability of entangled coherent states (ECS) which belongs to the class of non-Gaussian continuous variable entangled state. It is known that Reid criterion which is based on the inferred version of Heisenberg uncertainty relation well detects the steerability of Gaussian states such as two-mode squeezed vacuum. We find, however, that it cannot fully detect the steerability of ECS: it only reveals the steerability of even ECS of limited range of size and never does for the case of odd ECS. Therefore, we adopted a stronger criterion which is based on inferred entropic uncertainty relation and found that it can fully detect the steerability of ECSs of any size and parity. We have also examined the steering behavior of ECSs when they are subject to the vacuum noise. The steering range of time is slightly different depending on the initial size of ECS. As the concept of steering is asymmetric by its definition, there can be a possibility of unidirectional steering: one party can steer the other party's state but not vice versa. We have examined this one-way steering by allowing only one party of the state to undergo the vacuum noise. As a result, we find that there are circumstances that the non-decohered party can steer the decohered party but not vice versa. In the case of the vacuum noise, it might be a general phenomenon for every entangled state, which must be further investigated thoroughly.

ACKNOWLEDGMENTS

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