# Frozen quantum correlation of continuous variable system in non-Markovian reservoirs

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Protecting quantum correlation from decoherence is one of the focused issues in quantum information processing. It was commonly recognized that any initial quantum correlation of a composite systems decreases asymptotically or abruptly to zero due to the interactions of the subsystems with their local Markovian dissipative reservoirs. Here we show that this is not the case anymore for the continuous-variable bipartite system when the non-Markovian effect is taken in account. We find that a noticeable non-zero Gaussian quantum discord can be trapped in the steady state if each of the subsystems forms a localized mode with its local reservoir. The condition for this quantum discord frozen is given explicitly. The possible observation of our results in coupled cavity array system formed by photonic crystal is also investigated.

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*Introduction.*—Quantum correlation plays an essential role in quantum information science. In the early days of quantum information, quantum correlation is characterized by entanglement, which is viewed as the main resource for quantum information processing [1]. It causes the dramatic speedup of quantum computer over its classical counterpart. Recently, it was found that entanglement is not the only reason to cause such speedup and the similar speedup can also be achieved in the so-called deterministic one-qubit quantum computation by use of the zero-entanglement states [2, 3]. It has been attributed to another measure of quantum correlation [4], i.e. quantum discord (QD) [5, 6]. QD is also proved playing necessary role in assisted optimal nonorthogonal state discrimination [7]. These results indicate that entanglement cannot exhaust quantum correlation and QD characterizes the quantumness of correlations more generally than entanglement.

QD is defined as the difference between the quantum versions of two classically equivalent expressions for mutual information, which denote, respectively, the total and classical correlations [5]. Reflecting physically the information one can extracts on one subsystem by the local measurement to the other one, the classical correlation depends on the eigenbasis of the performed measurement. Then it is quantified by the maximal information over all possible measurement basis. Due to this optimization, the classical correlation as well as the QD can only be evaluated analytically for very limited cases. It has been proved that only for the two-qubit Bell-diagonal state [8, 9] and for the bipartite continuous-variable Gaussian state [10, 11], the QD can be evaluated analytically.

Thanks to these analytical achievements to quantify QD, the study of quantum correlations under decoherence, which is seen as an ubiquitous phenomenon in quantum world, attracts much attention in recent years. It is found theoretically [12–16] and experimentally [17] that, much different to the sudden death behavior of entanglement [18, 19], QD of a two-qubit system under individual decoherence decays to zero in asymptotical manner. The experimental [20] and theoretical [21, 22] works also show the similar results for the Gaussian QD of the continuous-variable system. It is also found both for discrete [23, 24] and continuous [25] variable systems that some QD can be developed transiently from certain initially classical state under a single local Markovian dissipative dynamics. It means that QD, in certain manner, is more robust than entanglement to decoherence. Another character of QD under decoherence much different to the behavior of entanglement is the presence of the frozen QD during the time evolution [17, 26, 27]. However, in all of the works, the QD decays exclusively to zero in the long-time limit. In quantum information processing, one always desires that the quantum correlation in certain initial state can be preserved in the long-time limit. Can we obtain finite frozen QD even in the steady state?

In this Letter, we propose to stabilize the Gaussian QD of a bipartite continuous-variable system by manipulating the non-Markovian effect of the reservoirs. By studying the correlation dynamics of the system, we reveal an explicit condition, under which a finite QD can be frozen in the steady state. We show that a coupled cavity array system realized in photonic crystal is a best candidate to observe this phenomenon. Our work indicates that in contrast to Markovian-approximation-based results, the non-Markovian effect has certain self-healing ability to the detrimental action of decoherence on quantum correlation. This gives a useful guide to decoherence control in quantum information science.

*Model and dynamics.*—Consider two noninteracting harmonic oscillators coupled to two independent reservoirs. The Hamiltonian of each local subsystem is ($\hbar = 1$)

$$\hat{H}^k = \omega_k \hat{a}_k^\dagger \hat{a}_k + \sum_l \omega_{kl} \hat{b}_{kl}^\dagger \hat{b}_{kl} + \sum_l (g_{kl} \hat{a}_k^\dagger \hat{b}_{kl} + \text{h.c.}), \quad (1)$$

where $\hat{a}_k$ and $\hat{b}_{kl}$ ($\hat{a}_k^\dagger$ and $\hat{b}_{kl}^\dagger$) are, respectively, the anni-

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hilation (creation) operators of the $k$-th harmonic oscillator with frequency $\omega_k$ and its corresponding reservoir. The coupling strength between them is given by $g_{kl}$. The system has an intense relevance in quantum-optical setting where the system oscillators can describe the quantized optical fields in cavity [28] or in circuit [29] QED, mechanical oscillators in opto-mechanics [30], and atomic ensemble under large-$N$ limit [31]. Currently, most quantum optical experiments are performed at low temperatures and under vacuum condition. Thus, we take the reservoirs to be at zero temperature in this work.

The exact decoherence dynamics of the system can be derived by Feynman and Vernon's influence-functional theory [32, 33]. The reduced density matrix of the system expressed in the coherent-state representation is given by

$$\rho(\bar{\boldsymbol{\alpha}}_f,\boldsymbol{\alpha}'_f;t)=\int d\mu(\boldsymbol{\alpha}_i)d\mu(\boldsymbol{\alpha}'_i)\mathcal{J}(\bar{\boldsymbol{\alpha}}_f,\boldsymbol{\alpha}'_f;t|\bar{\boldsymbol{\alpha}}_i,\boldsymbol{\alpha}'_i;0)\times\rho(\bar{\boldsymbol{\alpha}}_i,\boldsymbol{\alpha}'_i;0), \tag{2}$$

where $\mathcal{J}(\bar{\boldsymbol{\alpha}}_f,\boldsymbol{\alpha}'_f;t|\bar{\boldsymbol{\alpha}}_i,\boldsymbol{\alpha}'_i;0)$ is the propagating function. The coherent-state representation is defined as $|\boldsymbol{\alpha}\rangle=\prod_{k=1}^{2}\exp(\alpha_k a_k^\dagger)|0_k\rangle$, which are the eigenstates of annihilation operators and obey the resolution of identity, $\int d\mu(\boldsymbol{\alpha})|\boldsymbol{\alpha}\rangle\langle\boldsymbol{\alpha}|=1$ with the integration measures defined as $d\mu(\boldsymbol{\alpha})=\prod_k e^{-\bar{\alpha}_k\alpha_k}\frac{d\bar{\alpha}_k d\alpha_k}{2\pi i}$. $\bar{\boldsymbol{\alpha}}$ denotes the complex conjugate of $\boldsymbol{\alpha}$. $\mathcal{J}(\bar{\boldsymbol{\alpha}}_f,\boldsymbol{\alpha}'_f;t|\bar{\boldsymbol{\alpha}}_i,\boldsymbol{\alpha}'_i;0)$ is expressed as the path integral governed by an effective action which consists of the free actions of the forward and backward propagators of the system and the influence functional obtained from the integration of reservoir degrees of freedom. After evaluating the path integral, we get

$$\mathcal{J}(\bar{\boldsymbol{\alpha}}_f,\boldsymbol{\alpha}'_f;t|\bar{\boldsymbol{\alpha}}_i,\boldsymbol{\alpha}'_i;0)=\exp\Big\{\sum_{k=1,2}\big[u_k(t)\bar{\alpha}_{kf}\alpha_{ki}+\bar{u}_k(t)\bar{\alpha}'_{ki}\alpha'_{kf}+[1-|u_k(t)|^2]\bar{\alpha}'_{ki}\alpha_{ki}\big]\Big\}, \tag{3}$$

where $u_k(t)$ satisfies

$$\dot{u}_k(t)+i\omega_k u_k(t)+\int_0^t f_k(t-\tau)u_k(\tau)d\tau=0 \tag{4}$$

with $f_k(x)\equiv\int J_k(\omega)e^{-i\omega x}d\omega$ under the continuous limit of the environmental modes. Combining Eq. (3), the time-dependent state can be obtained from any initial state by evaluating the integration in Eq. (2). The exact decoherence dynamics, determined by Eq. (4), essentially depends on the so-called spectral density $J_k(\omega)\equiv\sum_l|g_{kl}|^2\delta(\omega-\omega_k)$, which characterizes the coupling strength of the different environmental modes to the system with respect to their frequencies. In the continuum limit it takes the form

$$J_k(\omega)=\eta_k\omega\Big(\frac{\omega}{\omega_c}\Big)^{n-1}e^{-\frac{\omega}{\omega_c}}, \tag{5}$$

where $\omega_c$ is a cutoff frequency, and $\eta_k$ is a dimensionless coupling constant. The environment is classified as Ohmic if $n=1$, sub-Ohmic if $0<n<1$, and super-Ohmic for $n>1$ [34]. Different spectral densities manifest different non-Markovian decoherence dynamics.

To compare with the conventional Born-Markovian approximate description to such system, we now derive a master equation by taking the time derivative to Eq. (2). We obtain the master equation as

$$\dot{\rho}(t)=\sum_{k=1,2}\{-i\Omega_k(t)[\hat{a}_k^\dagger\hat{a}_k,\rho(t)]+\Gamma_k(t)[2\hat{a}_k\rho(t)\hat{a}_k^\dagger-\hat{a}_k^\dagger\hat{a}_k\rho(t)-\rho(t)\hat{a}_k^\dagger\hat{a}_k]\}, \tag{6}$$

where

$$\Gamma_k(t)+i\Omega_k(t)\equiv-\dot{u}_k(t)/u_k(t). \tag{7}$$

It can be found that Eq. (6) keeps the Lindblad form but with time-dependent shifted frequency $\Omega_k(t)$ and decay rate $\Gamma_k(t)$. All the backactions induced by the non-Markovian effect have been incorporated into these time-dependent coefficients self-consistently.

*Dynamical frozen of Gaussian QD.*—Consider explicitly the initial state of the system as two-mode squeezed state $|\psi(0)\rangle=\exp[r(\hat{a}_1\hat{a}_2-\hat{a}_1^\dagger\hat{a}_2^\dagger)]|00\rangle$ with $r$ being the squeezing parameter. The time evolution of such state under Eq. (2) keeps the Gaussianity. The Gaussian state can be fully characterised by the covariance matrix $\sigma_{12}=\begin{pmatrix}\alpha_1 & \gamma\\ \gamma^T & \alpha_2\end{pmatrix}$, where $\alpha_k$ are the $2\times 2$ covariance matrices for the $k$-th subsystems, and $\gamma$ is the matrix containing the correlations between $(x_1,p_1)$ and $(x_2,p_2)$ with $\hat{x}_k=\frac{\hat{a}_k+\hat{a}_k^\dagger}{\sqrt{2}}$ and $\hat{p}_k=\frac{\hat{a}_k-\hat{a}_k^\dagger}{\sqrt{2}i}$. $\sigma_{12}$ can be easily estimated experimentally from the homodyne measurements to the amplitude quadratures $\hat{x}_k$ and $\hat{p}_k$. The QD for Gaussian state can be calculated as follows. The total correlation for a bipartite system is given by the mutual information $\mathcal{I}(\rho)=S(\rho_1)+S(\rho_2)-S(\rho)$, where $S$ is the von Neumann entropy and $\rho_{1(2)}$ is the reduced density matrix of the 1 (2) subsystem. Another measure of mutual information that only quantifies the amount of classical correlations extractable by a Gaussian measurement is $\mathcal{C}_1(\rho)=S(\rho_1)-\inf_{\sigma_M}S(\rho_{1|\sigma_M})$ where $\sigma_M$ is the covariance matrix of the measurement on mode 2. As it only captures the classical correlations, the difference, $\mathcal{D}_1=\mathcal{I}(\rho)-\mathcal{C}_1(\rho)$, is a measure of Gaussian quantum correlation that is coined Gaussian QD. An explicit expression for this QD has been found [11]:

$$\mathcal{D}(\sigma_{12})=\mathfrak{f}(\sqrt{I_2})-\mathfrak{f}(\nu_-)-\mathfrak{f}(\nu_+)+\mathfrak{f}(\sqrt{m}) \tag{8}$$

with $\mathfrak{f}(x)=(\frac{x+1}{2})\log\frac{x+1}{2}-(\frac{x-1}{2})\log\frac{x-1}{2}$ and

$$m=\begin{cases}\frac{2I_3^2+(I_2-1)(I_4-I_1)+2|I_3|\sqrt{I_3^2+(I_2-1)(I_4-I_1)}}{(I_2-1)^2}, & \text{a)}\\ \frac{I_1I_2-I_3^2+I_4-\sqrt{I_3^4+(I_4-I_1I_2)^2-2C^2(I_4+I_1I_2)}}{2I_2}, & \text{b)}\end{cases} \tag{9}$$

where a) applies if $(I_4-I_1I_2)^2\le I_3^2(I_2+1)(I_1+I_4)$ and b) applies otherwise. $I_k=\det\alpha_k$, $I_3=\det\gamma$, $I_4=\det\sigma_{12}$

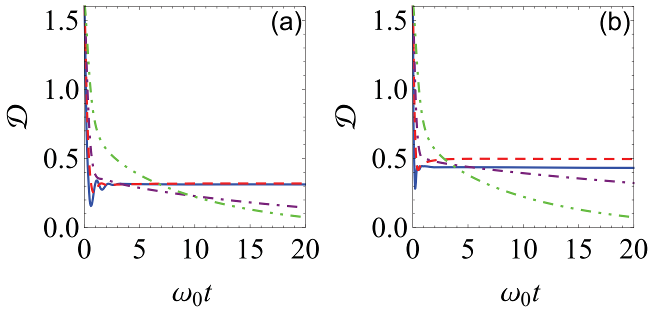


FIG. 1. (Color online) Time evolution of Gaussian quantum discord for super-Ohmic spectral density when (a): $\omega_c/\omega_0 = 1.0$ and $\eta = 0.08$ (dot-dot-dashed), 0.25 (dot-dashed), 0.5 (dashed), and 1 (solid); and when (b): $\eta = 0.08$ and $\omega_c/\omega_0 = 1.0$ (dot-dot-dashed), 1.5 (dot-dashed), 2.0 (dashed), and 3.0 (solid). The squeezing parameter is chosen as $r = 1.0$.

are the symplectic invariants and $\nu_\pm^2 = \frac{1}{2}(\delta \pm \sqrt{\delta^2 - 4I_4})$ with $\delta = I_1 + I_2 + 2I_3$ are the symplectic eigenvalues.

Choosing the super-Ohmic spectral density, explicitly $n = 3$ in Eq. (5), as an example, we plot in Fig. 1 the time evolution of the Gaussian QD for the initial two-mode squeezed state. Compared with the Ohmic and sub-Ohmic spectral densities, the super-Ohmic one is higher-frequency dominate, which will cause a strong modification to the short-time decoherence dynamics of the system. It has been shown that the super-Ohmic spectral density can describe the phonon bath in one or three dimensions, depending on the symmetry properties of the strain field [35] and a charged particle coupled to its own electromagnetic field [36]. We can see from Fig. 1(a) that the Gaussian QD decays to zero when the coupling is weak, which is qualitatively consistent with the results under Born-Markovian approximation. However, it is remarkable to find that the Gaussian QD tends to be frozen partially in the steady state with the increase of the coupling constant. This is dramatically contrary to one's expectation that a stronger coupling between the system and the environment always induces a severer decoherence to the system. The similar Gaussian QD frozen can also be achieved with the increase of the cutoff frequency in Fig. 1(b).

We argue that the formation of a localized mode between each of the harmonic oscillators and its local reservoir plays essential role in this Gaussian QD frozen. To verify this, we perform a Fourier transform to Eq. (4) and obtain

$$y(E) \equiv \omega_0 - \int_0^\infty \frac{J(\omega)}{\omega - E} d\omega = E. \tag{10}$$

Combining with Eq. (5), we can find that $y(E)$ is a monotonically decreasing function in the region $E \in (-\infty, 0)$. It means that Eq. (10) may have one and only one negative root if the system parameters fulfill $y(0) < 0$. On the other hand, no further discrete root exists in the region $(0, +\infty)$ because that would make the integration in $y(E)$ divergent. After the inverse Fourier transform,

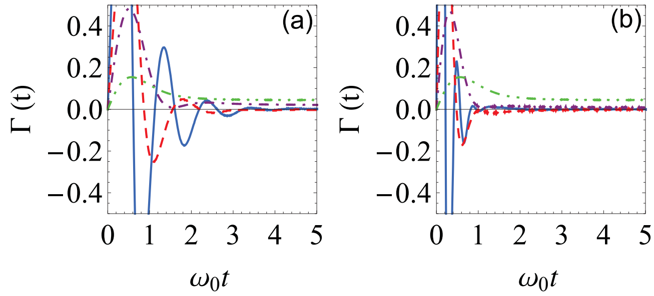


FIG. 2. (Color online) The corresponding decay rate of Fig. 1(a) and (b). The localized mode is formed when $\eta > 0.5$ for (a) and $\omega_c > 1.84\omega_0$ for (b).

the obtained $u_k(t)$ contributed from this discrete negative root will have a vanishing decay rate $\Gamma_k(t)$ according to Eq. (7). This vanishing decay rate causes the decoherence inhabited to the system. It means that the discrete negative root for Eq. (10) actually corresponds to a stationary state to Eq. (4), which preserves the quantum coherence in its superposed components during time evolution. We call this stationary state the localized mode of the whole system [37]. For our super-Ohmic spectral density, we can readily evaluated that the localized mode is formed when $\omega_0 - 2\eta\frac{\omega_c^3}{\omega_0^2} <$ is fulfilled. This criterion gives a basic judgement on the condition under which the Gaussian QD frozen is present.

To verify dynamical consequence of the formed localized mode, we plot in Fig. 2 the decay rate in the case Fig. 1(a,b). We can see that if the localized mode is absent, the decay rate keeps to be positive and tends to a positive value, which, as expected, will induces monotonic decoherence to the system (as shown in Fig. 1). On the contrary, if the localized mode is present, the decay rate is transiently negative, which manifests the lost information/energy of the system returns back from the reservoir back. Another character different to the case when the localized mode is absent is that the decay rate tends to zero asymptotically. This vanishing decay rate causes the decoherence of the system ceased in the long-time limit. This give an explanation why a strong coupling can induce a suppressed decoherence in Fig. 1.

From above analysis, we can conclude that the Gaussian QD frozen is present due to an interplay between the formed localized mode and the non-Markovian effect. The localized mode provides an ability to froze the Gaussian QD, while the non-Markovian effect provides a dynamical way to froze the Gaussian QD. The mechanism of the stable Gaussian QD frozen in our system is linked to the non-Markovian memory effect of the harmonic oscillator with its local reservoir when the localized mode is formed. It is much different to the case of two harmonic oscillators coupled to a comment reservoir [38], where a stable QD is established due to an indirect interaction between the two harmonic oscillators induced effectively by the common reservoir.

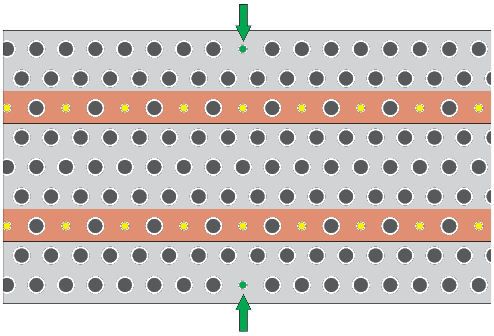

FIG. 3. (Color online) Two initially correlated cavity fields propagating in two cavity arrays formed in photonic crystal platform.

*Physical realization.*— With the basic criterion at hands, we can see that the Gaussian QD frozen we revealed is a generic phenomenon in open quantum systems. A best candidates to observe our prediction is the system of two chains of coupled cavity arrays, which can now be realized experimentally in micro-disc cavities coupled by one tapered optical fiber [39], in photonic crystal system [40–42], and synthesized in optical waveguide array system [43, 44]. In Fig. 3 we depict the schematic illustration to this scheme realized in photonic crystal system. Here two initially correlated quantized optical fields are fed into the two system cavities. With some probability the fields in the two system cavities will hop respectively to the two spatially separated coupled cavity arrays. Each of the local system is governed by $\hat{H}^{(1)} = \omega_0\hat{a}^\dagger\hat{a} + \omega_C\sum_{j=0}^{N-1}\hat{b}_j^\dagger\hat{b}_j + (g\hat{a}^\dagger\hat{b}_0 + \xi\sum_{j=0}^{N-2}\hat{b}_{j+1}^\dagger\hat{b}_j + \text{h.c.})$. A Fourier transform $\hat{b}_j = \sum_k \hat{b}_k e^{ikjx_0}$ can recast $\hat{H}^{(1)}$ into

$$\hat{H}^{(1)} = \omega_0\hat{a}^\dagger\hat{a} + \sum_k \epsilon_k\hat{b}_k^\dagger\hat{b}_k + \frac{g}{\sqrt{N}}\sum_k(\hat{a}^\dagger\hat{b}_k + h.c.) \quad (11)$$

with $\epsilon_k = \omega_C + 2\xi\cos kx_0$ and $x_0$ being the spatial separation between the two neighbour cavities of the cavity arrays. One can notice that the dispersion relation of the field in such structured reservoirs shows finite band width, which can induce a strong non-Markovian even in the weak and intermediate coupling regimes.

In Fig. 4(a), we plot the possible formation of the localized mode manifested by the intersection points between the dotted line with each line in different parameter regimes. It can be seen that if there is no intersection point, which means the localized mode is absent, then the Gaussian QD, as shown in Fig. 4(b), decays to zero. Whenever the localized mode is formed, certain finite Gaussian QD can be frozen in the steady state. An interesting observation in this situation is that a large amount of Gaussian QD can be frozen even there is no strong coupling between the system with the reservoirs. This reduces greatly the experimental difficult in the practical realization.

*Conclusions.*—In summary, focusing on how to protect the quantum correlation from decoherence, we have

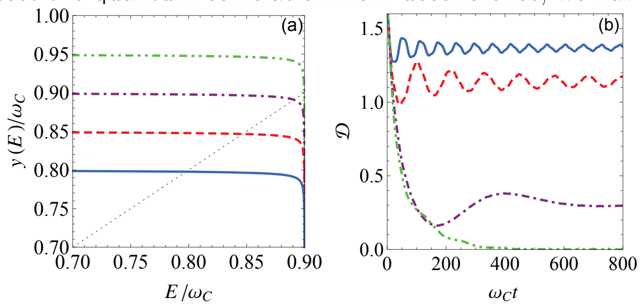


FIG. 4. (Color online) (a): The formation of a localized mode manifested by intersection point of the thin dotted line with the thick lines in different parameter regime. (b) The corresponding evolution of Gaussian QD. $\xi = 0.05\omega_C$, $g = 0.02\omega_C$, $N = 200$ and $\omega_0 = 0.95\omega_c$ (dot-dot-dashed green), $0.9\omega_c$ (dot-dashed purple), $0.85\omega_c$ (dashed red), and $0.8\omega_c$ (solid blue).

studied the dynamics of Gaussian QD of two harmonic oscillators interacting with two independent reservoirs. We have revealed a physical mechanism under which the decoherence of Gaussioan QD can be avoided and a finite Gaussian QD can be frozen in the steady state. We found that it is the interplay between a formed localized mode and the non-Markovian effect which plays essential role in the Gaussian QD frozen. The possible observation of our prediction in coupled cavity array system realized in photonic crystal platform has also been investigated. Our result suggests a control way to beat the effect of decoherence by engineering the spectrum of the reservoirs to approach the non-Markovian regime and to form localized mode of the whole system. This can be readily realized in the newly emerged field, i.e. reservoir engineering [45, 46], on controlling the quantum system by tailoring its coupling to the reservoirs.

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## Supplementary material

*The covariance matrix.*—The initial state can be represented in the coherent-state representation as

$$\rho(\bar{\boldsymbol{\alpha}}_i, \boldsymbol{\alpha}'_i; 0) = \frac{\exp[-\tanh r(\bar{\alpha}_{1i}\bar{\alpha}_{2i} + \alpha'_{1i}\alpha'_{2i})]}{\cosh^2 r}. \quad (12)$$

Substituting Eq. (12) into Eq. (2), we can obtain the evolved state as

$$\rho(\bar{\boldsymbol{\alpha}}_f, \boldsymbol{\alpha}'_f; t) = a\exp[\sum_{k\neq k'}(\frac{b}{2}\bar{\alpha}_{kf}\bar{\alpha}_{k'f} + c\bar{\alpha}_{kf}\alpha'_{kf} + \frac{b^*}{2}\alpha'_{kf}\alpha'_{k'f})], \quad (13)$$

where

$$a = \frac{1}{\cosh^2|r|\,[1-\tanh^2|r|\,(1-|u(t)|^2)^2]}, \quad (14)$$

$$b = \frac{-\tanh|r|\,u(t)^2}{1-\tanh^2|r|\,(1-|u(t)|^2)^2}, \quad (15)$$

$$c = \frac{\tanh^2|r|\,(1-|u(t)|^2)\,|u(t)|^2}{1-\tanh^2|r|\,(1-|u(t)|^2)^2}. \quad (16)$$

For the continuous-variable (Gaussian-type) bipartite state, its density matrix is characterized by the covariance matrix defined as the second moments of the quadrature vector $\hat{X} = (\hat{x}_1, \hat{p}_1, \hat{x}_2, \hat{p}_2)$,

$$\sigma_{ij} = \langle \Delta\hat{X}_i\Delta\hat{X}_j + \Delta\hat{X}_j\Delta\hat{X}_i\rangle, \quad (17)$$

where $\Delta\hat{X}_i = \hat{X}_i - \langle\hat{X}_i\rangle$, and $\hat{x}_i = \frac{\hat{a}_i+\hat{a}_i^\dagger}{\sqrt{2}}$, $\hat{p}_i = \frac{\hat{a}_i-\hat{a}_i^\dagger}{i\sqrt{2}}$. From the time-dependent state (13), the covariance matrix for the harmonic oscillators can be calculated straightforwardly,

$$\sigma = 2\begin{pmatrix} \frac{y(1+d)}{2(1-d)^2} & 0 & \frac{a\mathrm{Re}[b]}{x} & \frac{a\mathrm{Im}[b]}{x} \\ 0 & \frac{y(1+d)}{2(1-d)^2} & \frac{a\mathrm{Im}[b]}{x} & \frac{-a\mathrm{Re}[b]}{x} \\ \frac{a\mathrm{Re}[b]}{x} & \frac{a\mathrm{Im}[b]}{x} & \frac{y(1+d)}{2(1-d)^2} & 0 \\ \frac{a\mathrm{Im}[b]}{x} & \frac{-a\mathrm{Re}[b]}{x} & 0 & \frac{y(1+d)}{2(1-d)^2} \end{pmatrix}, \quad (18)$$

where $x = [(1-c)^2 - |b|^2]^2$, $y = \frac{a}{1-c}$, and $d = c + \frac{|b|^2}{1-c}$.

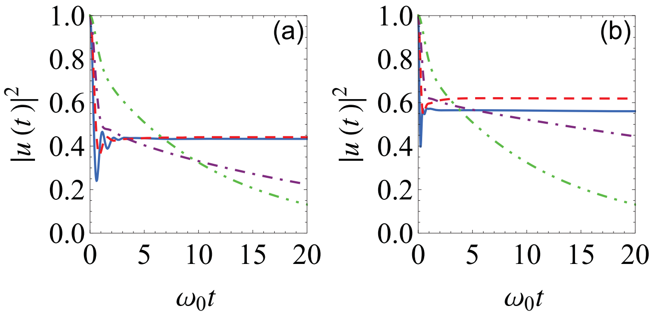


FIG. 5. (Color online) The corresponding $|u(t)|^2$ of Fig. 1(a) and (b). The localized mode is formed when $\eta > 0.5$ for (a) and $\omega_c > 1.84\omega_0$ for (b).

*Anomalous decoherence.*—Accompanying with the formation of the localized mode of the whole system, the dynamics of the reduced system is inhibited. This can be verified by the time-dependent behaviors of $u(t)$. In Fig. 5, we plot the evolution of $|u(t)|^2$ corresponding to the parameter regimes used in Fig. 1(a,b), respectively. We can see that with the formation of the localized mode above the critical point $\eta = 0.5$ for Fig. 5(a) and $\omega_c = 1.84\omega_0$ for Fig. 5(b), the time-dependent behavior of $|u(t)|^2$ shows qualitatively changes. If the localized mode is absent, $|u(t)|^2$ decays to zero monotonically, which is consistent with the results under Born-Markovian approximation. On the other hand, if the localized mode is present, $|u(t)|^2$ tends to a finite value after transient oscillation. It indicates the ceasing of the decoherence in the long-time limit, which also consistent with the vanishing decay rate in Fig. 2. It deviates qualitatively the results under Born-Markovian approximation. This shows clearly that the non-Markovian effect can not only induce transient oscillation, but also induce dramatic change to the steady state behavior to the open quantum system. Equipped with this anomalous decoherence, it is not hard to understand the Gaussian QD frozen revealed in our work.