Communication in non-inertial frames

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The dynamics of entangled two qubit pairs in non -inertial frames is investigated. The degree of entanglement is quantified for different classes of travelling states. The accelerated channels are used as quantum channels to teleport unknown accelerated information. The possibility of using the accelerated channels to perform quantum coding is discussed, where it is shown that the coding information in partial entangled states is more robust than using maximum entangled states.

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I. INTRODUCTION

Investigating the behavior of entanglement in non-inertial frames represents a cornerstone on relativistic quantum information (see for example Alsing and Milburn[1, 2]). Some efforts has been done in this direction for understanding entanglement in non-inertial frames. As an example, the dynamics of entanglement in non-inertial frames for system initially prepared in *maximum entangled* state is investigated by Alsing et. al. [3], where it is shown that the entanglement is degraded by the Unruh effect [4] and its lower limit is enough for quantum teleportation. The dynamics of the mutual information is investigated by Landulfo and Matsas [5, 6] in non-inertial frames. Ramzan and Khan have investigated the decoherence and entanglement degradation of a qubit-qutrit system in non-inertial frames[7]. The classical and quantum correlations of scalar field in the inflationary universe have been discussed by Nambu and Ohsumi [8].

In this paper, some of our recent results are reviewed [9–11] in context of relativistic quantum information. The dynamics of a general two qubit state in Rindler spaces are introduced, where only the density operator in the first region of Rindler's space is reported,(for more detials see [9]). The degree of entanglement is quantified for different initial states setting. The possibility of using the accelerated channel to teleport accelerate or non-accelerated is discussed extensively in Metwally [10]. Finally, the behavior of coded and decoded information for systems prepared initially in a maximum or partial entangled state are briefly reported[11].

The paper is arranged as follows. In Sec. II, the system and its evolution in the Rindler space are described. Entanglement is quantified in Sec. III for different initial states. Sec.IV is devoted to quantum teleportation protocol in non-inertial frames. Quantum coding using the accelerated channels is discussed in Sec. V. Finally, the results are summerized in Sec. VI.

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II. THE MODEL

The suggested model consists of a two qubits state. Each qubit is accelerated with a different acceleration on the same frame. A general two qubit system can be written as[12, 13],

$$\rho_{ab} = \frac{1}{4}(1 + \vec{s}_a \cdot \overset{\downarrow}{\sigma}_a + \vec{s}_b \cdot \overset{\downarrow}{\sigma}_b + \vec{\sigma}_a \cdot \overset{\downarrow\longrightarrow}{C} \cdot \overset{\downarrow}{\sigma}_b), \quad (1)$$

where $\vec{\sigma}_a = (\sigma_{ax}, \sigma_{ay}, \sigma_{az})$ and $\vec{\sigma}b = (\sigma_{bx}, \sigma_{by}, \sigma_{bz})$ are the Pauli matrices for Alice and Rob's qubit respectively, $\vec{s}_a = (s_{ax}, s_{ay}, s_{az})$ with $s_{ai} = tr\{\rho_{ab}\sigma_{ai}\}$, while $\vec{s}_b = (s_{bx}, s_{by}, s_{bz})$, $s_b = tr\{\rho_{ab}\sigma_{bi}\}$ are Bloch vectors for both qubits respectively. The dyadic $\overset{\downarrow\longrightarrow}{C}$ is a 3×3 matrix with elements are defined by $c_{ij} = tr\{\rho_{ab}\sigma_{ai}\sigma_{bi}\}$. The state (1) represents a quantum channel between the two users in the Minkowski frame. For accelerated particles, the Rindler coordinates are more convenient.

To find the form of the Minkowski state in terms of Rindler' space, one has to use Bogoliubov transformation,

$$\begin{aligned} \nu_k &= \cos r \mathcal{A}_k^I - e^{i\phi} \sin r \mathcal{B}_{-k}^{II}, \\ \mu_k^\dagger &= e^{i\phi} \sin r \mathcal{A}_k^I + \cos r \mathcal{B}_{-k}^{II_+}, \end{aligned} \quad (2)$$

where $\mathcal{A}_k^I$ and $\mathcal{B}_{-k}^{II_+}$ represent the annihilation and creation operators in the regions I and II respectively. The parameter ϕ is an unimportant phase that can be absorbed into the definition of the operators. In terms of Rindler Fock states, the fermions vacuum state $|0_k\rangle$ and the one particle state $|1_k\rangle$ take the form [3]

$$\begin{aligned} |0_k\rangle &= C_i |0_k\rangle_I |0_k\rangle_{II} + S_i |1_k\rangle_I |1_k\rangle_{II}, \\ |1_k\rangle &= |1_k\rangle_I |0_k\rangle'_{II} \end{aligned} \quad (3)$$

where it is assumed that the phase ϕ is absorbed into the definition of the operators, $\mathcal{C}_i = \cos r_i, \mathcal{S}_i = \sin r_i$, with $\tan r_i = e^{-\pi\omega_i \frac{c}{a_i}}$, a_i is the acceleration, ω_i is the frequency of the travelling qubits, c is the speed of light, and $i = A, R$ stand for Alice and Rob respectively. This effect is called Unruh's effect (3), where it is assumed that the phase ϕ is absorbed into the definition of the operators. In what follows we shall consider some important classes of entangled channels and investigate their behavior in non-inertial frames.

By using Eqs.(1)and (3) and tracing out the inaccessible modes in the region II, the state of Alice-Rob in the region I is given by[9],

$$\rho_{\tilde{A}_I\tilde{R}_I} = \begin{pmatrix} \varrho_{11}\mathcal{C}_1^2\mathcal{C}_2^2 & \varrho_{12}\mathcal{C}_1^2\mathcal{C}_2 & \varrho_{13}\mathcal{C}_1\mathcal{C}_2^2 & \varrho_{14}\mathcal{C}_1\mathcal{C}_2 \\ \varrho_{21}\mathcal{C}_1^2\mathcal{C}_2 & \mathcal{C}_1^2(\varrho_{22}+\varrho_{11}\mathcal{S}_2^2) & \varrho_{23}\mathcal{C}_1\mathcal{C}_2 & \varrho_{24}\mathcal{C}_1 \\ \varrho_{31}\mathcal{C}_1\mathcal{C}_2 & \varrho_{32}\mathcal{C}_1\mathcal{C}_2 & (\varrho_{33}+\varrho_{11}\mathcal{S}_1^2)\mathcal{C}_2^2 & (\varrho_{34}+\varrho_{12}\mathcal{S}_1^2)\mathcal{C}_2 \\ \varrho_{41}\mathcal{C}_1\mathcal{C}_2 & (\varrho_{42}+\varrho_{31}\mathcal{S}_2^2)\mathcal{C}_1 & (\varrho_{43}+\varrho_{21}\mathcal{S}_1^2)\mathcal{C}_2 & \varrho_{44}+\varrho_{33}\mathcal{S}_2^2+\mathcal{S}_1^2(\varrho_{22}+\varrho_{11}\mathcal{S}_2^2) \end{pmatrix}. \quad (4)$$

III. ENTANGLEMENT

In this section, the entanglement behavior for different classes of initial states settings is investigated. The earliest investigation has considered only one qubit moving with a uniform acceleration while the other one stays stationary [14]. In the current study, all different situations are extensively investigated. To measure the entanglement of the generated entangled channels, Wootters'concurrence [15] are used,

$$\mathcal{C} = \max\Big\{0, \lambda_1-\lambda_2-\lambda_3-\lambda_4\Big\}, \quad (5)$$

where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$ and λ_i are the eigenvalues of the density operator $\rho = \sigma_y\tau_y\rho^*\sigma_y\tau_y$, ρ^* is the complex conjugate of ρ.

Let us assume that we have a system prepared initially in a generalized Werner state which is defined as:

$$\rho_{AR} = \frac{1}{4}(1 + c_{xx}\sigma_x\tau_x + c_{yy}\sigma_y\tau_y + c_{zz}\sigma_z\tau_z). \quad (6)$$

From this class, one can obtain the maximum entangled states,. For example if we set $c_{xx} = c_{yy} = c_{zz} = -1$, we obtain the singlet state (see for example Metwally [16]). Also, if we set $c_{xx} = c_{yy} = c_{zz} = x$, one obtains the Werner state.

The concurrence $\mathcal{C}$ for a system initially prepared in maximum entangled state (MES), and Werner states with $x = -0.9, -0.8$ is shown in Fig.(1a). This figure displays the concurrence behavior of the generated entangled channel between Alice and Rob in the first region $\rho_{\tilde{A}_I\tilde{R}_I}$. It is clear that, since we start with MES, the concurrence $\mathcal{C} = 1$ at $r_a = r_b = 0$. However if, the first qubit is accelerated while the second qubit stays stationary i.e., $r_a = r$ and $r_b = 0$, then the concurrence decreases smoothly and gradually as depicted in Fig.(1b).

Finally, let us consider a class of pure states. This class is characterized by one parameter p, which is equal to the length of the Bloch vectors i.e., $|\vec{s}| = |\vec{t}| = p$. Mathematically it can be written as [13]

$$\rho_{\mathrm{p}} = \frac{1}{4}\Big(1 + p(\sigma_x - \tau_x) - \sigma_x\tau_x - \sqrt{1-p^2}(\sigma_y\tau_y + \sigma_z\tau_z)\Big), \quad (7)$$

where Bloch vectors and the non-zero elements of the cross dyadic are given by,

$$\vec{s} = (p,0,0), \quad \vec{t} = (-p,0,0), \quad c_{xx} = -1, \quad c_{yy} = c_{zz} = -\sqrt{1-p^2}. \quad (8)$$

Fig.(2a), shows the behavior of entanglement in the first region I, where the system is prepared in a pure state (7) described by $p = \frac{1}{\sqrt{2}}$. It is clear that as the accelerations increase the entanglement decays. However, if only one particle is accelerated, the rate of entanglement degradation is small. This rate of degradation increases if both particles are accelerated. The effect of the parameter p is depicted in Fig.(2b), where it shows that for smaller values of p, i.e., the initial state has a large degree of entanglement, the entanglement is much larger compared with those for larger p, where the initial state has a small degree of entanglement. Also, if the two particles are accelerated, the entanglement decays fast.

IV. TELEPORTATION

In this section our recent results are reviewed [10], where the generated entangled state has been used to perform quantum teleportation for unknown accelerated or non-accelerated information. As mentioned above, the maximum entangled channels and a pure state with large degree of entanglement could be used as a quantum channel to perform the original quantum teleportation [17].

Assume that a source supplies Alice with unknown information coded in Rob's state as:

$$\big|\psi_B\big\rangle = \alpha\big|0\big\rangle + \beta\big|1\big\rangle, \quad (9)$$

where $|\alpha|^2 + |\beta|^2 = 1$. It is assumed that Alice 's qubit and Rob's qubit are accelerated in the same frame with

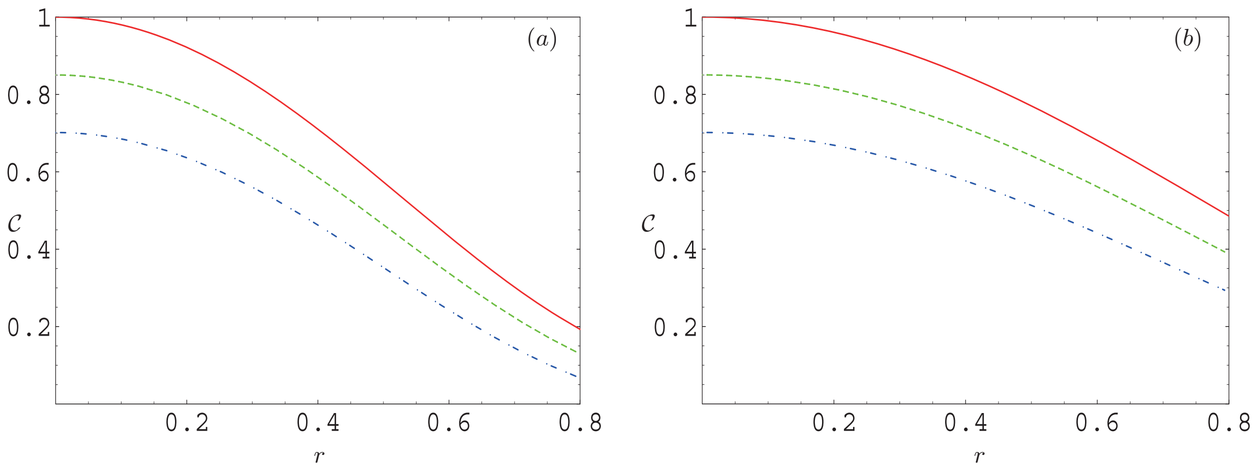

FIG. 1. The degree of entanglement in the first region I for a system prepared initially in maximum entangled state (solid curve), Werener state with $x = -0.9$ (dot-curve) and $x = -0.8$ dash (dot curve). It is assumed that (a) both particles are accelerated i.e, $r_a = r_b = r$ and (b) only the first qubit is accelerated.

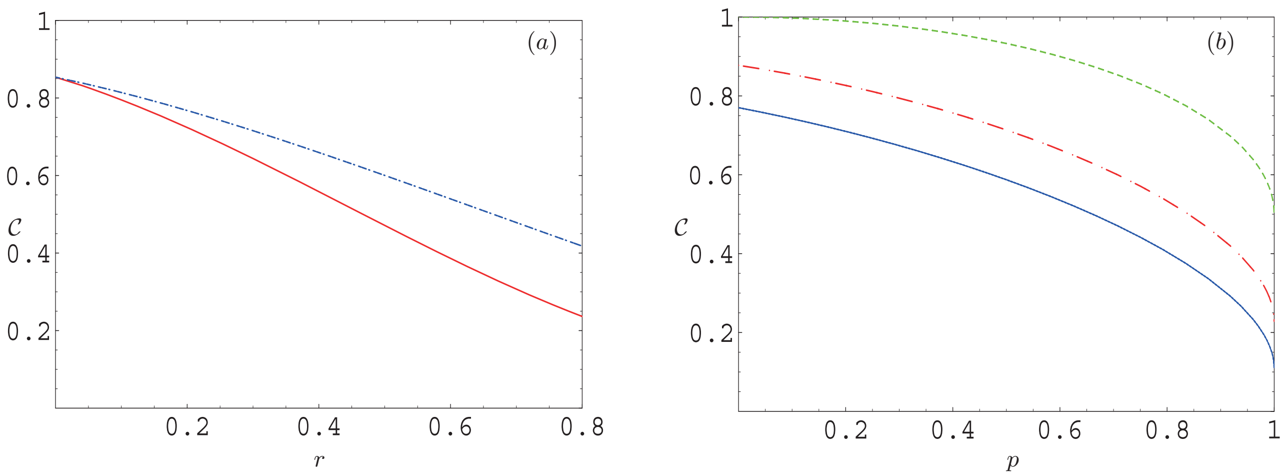

FIG. 2. The degree of entanglement in the first region I for a system prepared initially in a pure state (a) $p = 1/\sqrt{2}$ where for the dot curve it is assumed that only one particle is accelerated while the solid curve (the two qubits are accelerated (b) Represent the entanglement aganist the channel parameter p. The dot curve, dash-dot and solid curves for no acceleration, first particle is accelerated ($r_A = 0.5, r_R = 0$) and both particles are accelerated ($r_A = 0.5, r_R = 0.5$).

different accelerations. At rest, the frames of Alice and Bob are nearer so that Alice can make the measurements by using the basis on both frames [1].

Now let us assume that the coded information in the state (9) is accelerated according to Eq. (3), with an acceleration r_3. The density operator in the region I, which carries the unknown information ,is given by

$$\rho_B = |\alpha|^2 \cos^2 r_3 \big|0\big\rangle\big\langle 0\big| + |\alpha \sin r_3 + \beta|^2 \big|1\big\rangle\big\langle 1\big|. \qquad (10)$$

To teleport the unknown information which is coded on the state (10), Alice and Rob will use the accelerated state of the pure state in the region I as an quantum channel [9]. For this purpose the partners perform the original quantum teleportation protocol [10]. If Alice measures the state $\big|\phi_+\big\rangle = \frac{1}{\sqrt{2}}(\big|00\big\rangle + \big|11\big\rangle)$, then Rob will get this final state with a fidelity,

$$\mathcal{F}_{\mathcal{P}_a} = B_1\mu_{00} + B_2\mu_{11} \qquad (11)$$

where,

$$\begin{aligned} \mu_{00} &= \frac{1}{4}\Big((B_1 + B_2)(\varrho_{11} + \varrho_{33}) \\ &\quad + \frac{p(B_1 - B_2)}{4}\cos r_1(1 + \cos 2r_2)\Big), \\ \mu_{11} &= \frac{1}{4}\Big((B_1 + B_2)(\varrho_{22} + \varrho_{44}) \\ &\quad + \frac{p(B_1 - B_2)}{4}\cos r_1(3 - \cos 2r_2)\Big), \end{aligned} \qquad (12)$$

and

$$\varrho_{11} = \frac{1}{4}\Big[\frac{1-q}{4}(1+2\cos 2r_1) + \frac{1-q}{4}\cos 2r_2\Big],$$
$$\varrho_{22} = \frac{1}{16}\Big[q-1+2(q+1)\cos 2r_1 + (q+3)\cos 2r_2\Big],$$
$$\varrho_{33} = \frac{1}{4}\Big[\frac{1+q}{2}\cos 2r_1 + \frac{1+q}{4}(1+\cos 2r_2)\Big],$$
$$\varrho_{44} = \frac{1}{4}\Big[\frac{9-q}{4} - \frac{1+q}{2}\cos 2r_1 - \frac{3+q}{4}\cos 2r_2\Big], \quad (13)$$

and $B_1 = |\alpha|^2\cos^2 r_3$, $B_2 = |\alpha\sin r_3 + \beta|^2$, and $q = \sqrt{1-p^2}$.

Our results show that the fidelity of the teleported state depends on the initial degree of entanglement of the used quantum channel, the accelerations of the quantum channel, and the acceleration of teleported state. Also, sending classical information via quantum accelerated channel is much better than teleporting quantum information. For more discussion see Metwally[10].

V. QUANTUM CODING

In this section, the accelerated channel is used to send a coded information between the two users. For this purpose we consider a system of a two qubits prepared initially in $X-$state [18]. This class has been have been extensively used in the context of quantum information [19–21]. This system is characterized by

$$\vec{s}_a = \vec{s}_b = 0, \quad c_{ij} = 0 \text{ for } \quad i \neq j \text{ and } \quad c_{ij} \neq 0 \text{ for } \quad i = j. \quad (14)$$

Using (5),(6) and (20), we obtain the density operator of the accelerated channel in non-inertial frame [11]. However if, the states of Alice and Bob in the second region II are traced out, then the accelerated channel in the first region is given by

$$\rho_{ab}^{(I)} = \begin{pmatrix} \mathcal{B}_{11} & 0 & 0 & \mathcal{B}_{14} \\ 0 & \mathcal{B}_{22} & \mathcal{B}_{23} & 0 \\ 0 & \mathcal{B}_{32} & \mathcal{B}_{33} & 0 \\ \mathcal{B}_{41} & 0 & 0 & \mathcal{B}_{44} \end{pmatrix} \quad (15)$$

where

$$\begin{aligned} \mathcal{B}_{11} &= \mathcal{A}_{11}\cos^2 r_a \cos^2 r_b, \\ \mathcal{B}_{14} &= \mathcal{A}_{14}\cos r_a \cos r_b, \\ \mathcal{B}_{22} &= \cos^2 r_a(\mathcal{A}_{11}\sin^2 r_b + \mathcal{A}_{22}), \\ \mathcal{B}_{23} &= \mathcal{A}_{23}\cos r_a \cos r_b, \\ \mathcal{B}_{32} &= \mathcal{A}_{32}\cos r_a \cos r_b, \\ \mathcal{B}_{33} &= \cos^2 r_b(\mathcal{A}_{11}\sin^2 r_a + \mathcal{A}_{33}), \\ \mathcal{B}_{41} &= \mathcal{A}_{41}\cos r_a \cos r_b, \\ \mathcal{B}_{44} &= \sin^2 r_a(\mathcal{A}_{11}\sin^2 r_b + \mathcal{A}_{22}) \\ &\quad +\mathcal{A}_{33}\sin^2 r_b + \mathcal{A}_{44} \end{aligned} \quad (16)$$

where

$$\begin{aligned} \mathcal{A}_{11} &= \mathcal{A}_{44} = \frac{1}{4}(1+c_z), \\ \mathcal{A}_{22} &= \mathcal{A}_{33} = \frac{1}{4}(1-c_z), \\ \mathcal{A}_{23} &= \mathcal{A}_{32} = \frac{1}{4}(c_x+c_y), \\ \mathcal{A}_{14} &= \mathcal{A}_{41} = \frac{1}{4}(c_x-c_y). \end{aligned} \quad (17)$$

Now, we can use the output state (15) to perform the original quantum coding protocol [22–24]. At the end of the protocol, the maximum amount of information which Rob can extract from Alices message is bounded by,

$$I_R = -2(\lambda_+\log\lambda_+ + \lambda_-\log\lambda_-) - 2(\lambda_1\log\lambda_1 + \lambda_2\log\lambda_2 + \lambda_3\log\lambda_3 + \lambda_4\log\lambda_4) \quad (18)$$

where

$$\begin{aligned} \lambda_\pm &= \frac{1}{2}\Big\{1 + \frac{1}{2}\sqrt{\{(\mathcal{B}_{11}+\mathcal{B}_{33}) - (\mathcal{B}_{22}+\mathcal{B}_{44})\}^2 + 4(\mathcal{B}_{12}+\mathcal{B}_{34})(\mathcal{B}_{21}+\mathcal{B}_{43})}\ \Big\}, \\ \lambda_{1,2} &= \frac{1}{2}\Big\{\mathcal{B}_{11}+\mathcal{B}_{44}\Big\} \pm \sqrt{\left(\mathcal{B}_{11}-\mathcal{B}_{44}\right)^2 + 4\mathcal{B}_{41}\mathcal{B}_{14}}\Big\}, \\ \lambda_{3,4} &= \frac{1}{2}\Big\{\mathcal{B}_{22}+\mathcal{B}_{33}\Big\} \pm \sqrt{\left(\mathcal{B}_{22}-\mathcal{B}_{33}\right)^2 + 4\mathcal{B}_{23}\mathcal{B}_{32}}\Big\}, \end{aligned} \quad (19)$$

Fig.(3) shows the dynamics of the decoded information at Rob's hand, I_R which is coded in an accelerated channel in the first region (I). The dynamics of I_R information for a system prepared initially in a maximum entangled state is depicted in Fig. (3a), where it is assumed that both particles are accelerated. It is clear that at zero accelerations ($r_a = r_b = 0$), the amount of decoded information is large. Furthermore, as the accelerations increases, I_B decreases. However, the decreasing rate is larger when both particles are accelerated. The dynamics of decoded information for a system prepared initially in a partial entangled state is depicted in Fig.(3b), where we assume a class of Werner state. The behavior of I_R is similar to that shown in Fig.(3a), but the decreasing rate is smaller than that displayed in Fig.(3a). This shows that the coded information in a partial entangled state is much robust than that decoded in a maximum entangled state (see [11] for more discussion).

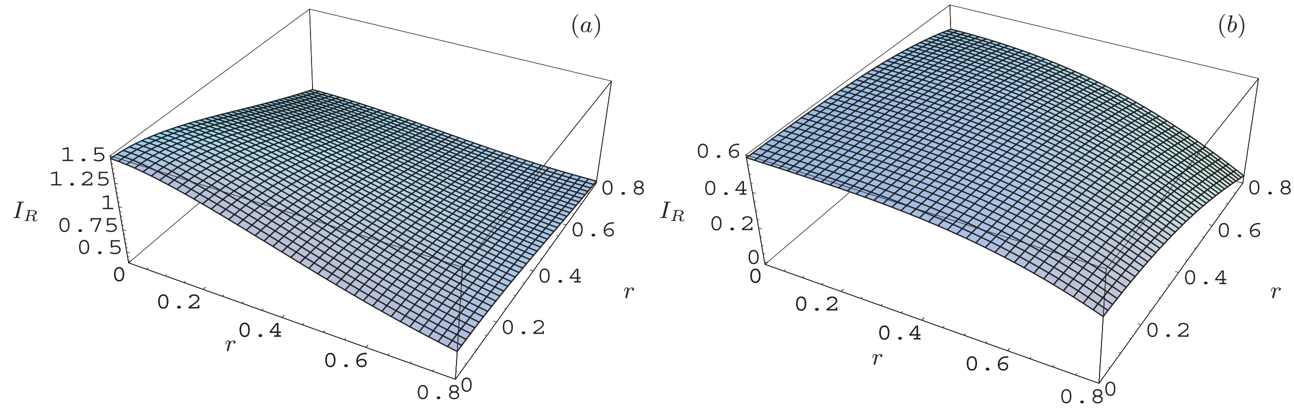

FIG. 3. Rob information I_R for a system prepared initially (a) Maximum entangled state (b) Werner state with $c_{xx} = c_{yy} = c_{zz} = 0.7$.

VI. CONCLUSION

In this paper, we review some of our recent results[9–11], where the dynamics of a general two qubit state is investigated in non-inertial frames. An analytical expression is obtained for the travelling state in both Rindler's region [9]. The degree of entanglement of the accelerated particles depends on their initial entanglement. The entanglement of pure state is more robust than that for maximum entangled state. If the two particles are accelerated, the degree of entanglement decreases fast. The possibility of using the output to teleport an accelerated information is discussed, where the fidelity of the telephoned state is evaluated analytically. Also, the output state is used as a quantum channel to send coded information. It is shown that the decoded information decreases as the acceleration increases. However, the lose of information coded in partial entangled states is smaller than that coded in maximum entangled state.

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